Q. 8. A transverse harmonic wave on a string is described by $y(x, t) = 3.0 \sin(36t + 0.018x + \pi/4)$, where x, y are in cm and t in sec. The positive direction of x-axis is from left to right.
(i) Is this a travelling or a stationary wave ? If it is travelling, what are the speed and direction of its propagation ? (ii) what are its amplitude and frequency ? (iii) what is the initial phase at the origin ? (iv) What is the least distance between two successive crests in the wave ?

Sol. Compare the given equation with that of a plane progressive wave of amplitude r, travelling with a velocity v from right to left :

$$y(x, t) = r \sin\left[\frac{2\pi}{\lambda}(vt + x) + \phi_0\right] \quad ...(i)$$

We find that (*i*) The given equation represents a transverse harmonic wave travelling from *right to left*. It is not a stationary wave.

(*ii*) The given eqn. can be rewritten as

$$y(x, t) = 3{\cdot}0 \sin\left[0{\cdot}018\left(\frac{36}{0{\cdot}018}t + x\right) + \frac{\pi}{4}\right] \quad ...(ii)$$

Equating coeffs of t in the two eqns. (*i*) and (*ii*), we get,

$$v = \frac{36}{0{\cdot}018} = 2000 \text{ cm/sec}$$

Obviously, $r = 3{\cdot}0$ cm.

Also, $\frac{2\pi}{\lambda} = 0{\cdot}018$

$$\lambda = \frac{2\pi}{0{\cdot}018} \text{ cm.}$$

Frequency, $\nu = \frac{v}{\lambda} = \frac{2000}{2\pi} \times 0{\cdot}018 = \mathbf{5{\cdot}73\ s^{-1}}$

(*iii*) Initial phase, $\phi_0 = \pi/4$ **radian**

(*iv*) Least distance between two successive crests of the wave = wavelength, $\lambda = \frac{2\pi}{0{\cdot}018}\text{cm} = \mathbf{349\ cm}$

Q. 9. For the wave described in Q.8 above, plot the displacement (y) versus (t) graphs for $x = 0$, 2 and 4 cm. What are the shapes of these graphs ? In which aspects does the oscillatory motion in travelling wave differ from one point to another : amplitude, frequency or phase ?

Sol. The transverse harmonic wave is

$$y(x, t) = 3{\cdot}0\left[36t + 0{\cdot}018x + \frac{\pi}{4}\right]$$

For $x = 0$,

$$y(0, t) = 3{\cdot}0 \sin[36t + \pi/4] \quad ...(i)$$

Here, $\omega = \frac{2\pi}{T} = 36$, $T = \frac{2\pi}{36} = \frac{\pi}{18}$ sec.

For different values of t, we calculate y using eqn. (*i*). These values are tabulated below :

t	0	$T/8$	$2T/8$	$3T/8$	$4T/8$	$5T/8$	$6T/8$	$7T/8$	T
y	$3/\sqrt{2}$	3	$3/\sqrt{2}$	0	$-3/\sqrt{2}$	-3	$-3/\sqrt{2}$	0	$3/\sqrt{2}$

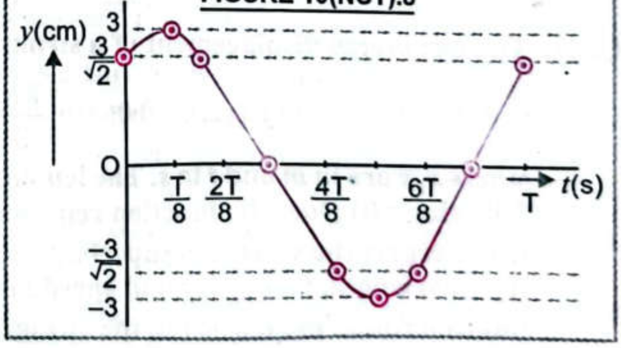


FIGURE 10(NCT).8

On plotting y versus t graph, we obtain a sinusoidal curve as shown in Fig. 10(NCT).8.
Similar graphs are obtained for $x = 2$ cm and $x = 4$ cm.
The oscillatory motion in travelling wave differs from one point to another only in terms of *phase*. Amplitude and frequency of oscillatory motion remain the same in all the three cases.

Q. 10. For the travelling harmonic wave, $y(x, t) = 2\cdot0 \cos 2\pi [10t - 0\cdot0080x + 0\cdot35]$, where x and y are in cm and t in s, what is the phase difference between oscillatory motion at two points separated by a distance of (i) 4 m (ii) 0·5 m (iii) $\lambda/2$ (iv) $3\lambda/4$?

Sol. The given equation can be rewritten as

$$y = 2\cdot0 \cos [2\pi (10t - 0\cdot0080x) + 2\pi \times 0\cdot35]$$

$$y = 2\cdot0 \cos \left[2\pi \times 0\cdot0080 \left(\frac{10t}{0\cdot0080} - x\right) + 0\cdot7\pi\right]$$

Compare it with the standard equation of a travelling harmonic wave, $y = r \cos \left[\frac{2\pi}{\lambda}(vt - x) + \phi_0\right]$

we get, $\frac{2\pi}{\lambda} = 2\pi \times 0\cdot0080$

Further, we know that for the path difference x, the phase diff., $\phi = \frac{2\pi}{\lambda} x$

(i) when, $x = 4\ m = 400\ cm$

$$\phi = \frac{2\pi}{\lambda} x = 2\pi \times 0\cdot0080 \times 400 = \mathbf{6\cdot4\pi\ rad}$$

(ii) when, $x = 0\cdot5 = 50\ cm$

$$\phi = \frac{2\pi}{\lambda} x = 2\pi \times 0\cdot0080 \times 50 = \mathbf{0\cdot8\pi\ rad}$$

(iii) when, $x = \lambda/2$

$$\phi = \frac{2\pi}{\lambda} \times \frac{\lambda}{2} = \mathbf{\pi\ rad}$$

(iv) when, $x = 3\lambda/4$

$$\phi = \frac{2\pi}{\lambda} \times \frac{3\lambda}{4} = \frac{3\pi}{2}\ \mathbf{rad}$$

Q. 11. The transverse displacement of a string (clamped at its two ends) is given by

$$y(x, t) = 0\cdot06 \sin \frac{2\pi}{3} x \cos (120\pi t)$$

where x, y are in m and t in s. The length of the string is 1.5 m and its mass is 3×10^{-2} kg. Answer the following : (i) Does the function represent a travelling or a stationary wave ?
(ii) Interpret the wave as a superimposition of two waves travelling in opposite directions. What are the wavelength, frequency and speed of propagation of each wave ?
(iii) Determine the tension in the string.

Sol. The given eqn. is $y(x, t) = 0\cdot06 \sin \frac{2\pi}{3} x \cos 120\pi t$ ...(*i*)

(*i*) As the equation involves harmonic functions of x and t separately, it represents a stationary wave.

(*ii*) We know that when a wave pulse $y_1 = r \sin \frac{2\pi}{\lambda}(vt - x)$

travelling along + direction of x-axis is superimposed by the reflected wave

$$y_2 = -r \sin \frac{2\pi}{\lambda}(vt - x)$$

travelling in opposite direction, a stationary wave

$$y = y_1 + y_2 = -2r \sin \frac{2\pi}{\lambda} x \cos \frac{2\pi}{\lambda} vt \text{ is formed.} \quad ...(ii)$$

Comparing (*i*) and (*ii*), we find that $\frac{2\pi}{\lambda} = \frac{2\pi}{3}$ or $\lambda = 3$ m

Also, $\frac{2\pi}{\lambda} v = 120\pi$ or $v = 60\,\lambda = 60 \times 3 = 180 \text{ ms}^{-1}$

Frequency, $\nu = \frac{v}{\lambda} = \frac{180}{3} = 60$ **hertz**

Note that both the waves have same wave length, same frequency and same speed.

(*c*) Velocity of transverse waves is $v = \sqrt{T/m}$ or $v^2 = T/m$

$T = v^2 \times m$, where $m = \frac{3 \times 10^{-2}}{1 \cdot 5} = 2 \times 10^{-2}$ kg/m

$\therefore \quad T = (180)^2 \times 2 \times 10^{-2} = \mathbf{648\ N}$

Q. 12. (i) For the wave on a string described in Q. 11 above, do all the points on the string oscillate with the same (a) frequency, (b) phase, (c) amplitude ? Explain your answers. (ii) What is the amplitude of a point 0·375 m away from one end ?

Sol. All the points on the string (*i*) have the same frequency except at the nodes (where frequency is zero) (*ii*) have the same phase everywhere in one loop except at the nodes.

However, the ampltiude of vibration at different points is different.

From $\quad y(x, t) = 0 \cdot 06 \sin\left(\frac{2\pi}{3} x\right) \cos(120\,\pi\, t)$,

the amplitude at $x (= 0 \cdot 375 \text{ m})$ is $= 0 \cdot 06 \sin \frac{2\pi}{3} x \times 1 = 0 \cdot 06 \sin \frac{2\pi}{3} \times 0 \cdot 375 = 0 \cdot 06 \sin \frac{\pi}{4} = \frac{0 \cdot 06}{\sqrt{2}}$

$= \mathbf{0 \cdot 042\ m}$

Q. 13. Given below are some functions of *x* and t to represent the displacement (transverse or longitudinal) of an elastic wave. State which of these represent (i) a travelling wave, (ii) a stationary wave or (iii) none at all ?

(a) $y = 2\cos(3x)\sin(10t)$ **(b)** $y = 2\sqrt{x - vt}$

(c) $y = 3\sin(5x - 0 \cdot 5t) + 4\cos(5x - 0 \cdot 5t)$ **(d)** $y = \cos x \sin t + \cos 2x \sin 2t$

Sol. (*a*) It represents a stationary wave as harmonic functions of *x* and *t* are contained separately in the equation.

(*b*) It cannot represent any type of wave.

(*c*) It represents a progressive/travelling harmonic wave.

(*d*) This equation is sum of two functions each representing a stationary wave. Therefore, it represents superposition of two stationary waves.

Q. 14. A wire stretched between two rigid supports vibrates in its fundamental mode with a frequency of 45 Hz. The mass of the wire is 3·5 × 10⁻² kg and its linear mass density is 4·0 × 10⁻² kg m⁻¹. What is (a) the speed of a transverse wave on the string, and (b) the tension in the string ?

Sol. Here, $\nu = 45$ Hz, $M = 3 \cdot 5 \times 10^{-2}$ kg ; mass/length $= m = 4 \cdot 0 \times 10^{-2} \text{ kg m}^{-1}$

$\therefore \quad l = \frac{M}{m} = \frac{3 \cdot 5 \times 10^{-2}}{4 \cdot 0 \times 10^{-2}} = \frac{7}{8}$ m

As, $\frac{\lambda}{2} = l = \frac{7}{8} \quad \therefore \quad \lambda = \frac{7}{4} m = 1 \cdot 75$ m.

The speed of the transverse wave $v = \nu\lambda = 45 \times 1 \cdot 75 = \mathbf{78 \cdot 75\ m/s}$

(*b*) As, $v = \sqrt{\frac{T}{m}} \quad \therefore \quad T = v^2 \times m = (78 \cdot 75)^2 \times 4 \cdot 0 \times 10^{-2} = \mathbf{248 \cdot 06\ N}$

Q. 15. A metre-long tube open at one end, with a movable piston at the other end, shows resonance with a fixed frequency source (a tuning fork of frequency 340 Hz) when the tube length is 25.5 cm or 79.3 cm. Estimate the speed of sound in air at the temperature of the experiment. The edge effects may be neglected.

Sol. As there is a piston at one end of the tube, it behaves as a closed organ pipe, which produces odd harmonics only. Therefore, the pipe is in resonance with the fundamental note and the third harmonic (79·3 cm is about 3 times 25·5 cm]

In the fundamental mode, $\frac{\lambda}{4} = l_1 = 25{\cdot}5$

$\lambda = 4 \times 25{\cdot}5 = 102 \text{ cm} = 1{\cdot}02 \text{ m}.$

Speed of sound in air, $v = \nu\lambda = 340 \times 1{\cdot}02 = \mathbf{346{\cdot}8 \text{ m/s}}$

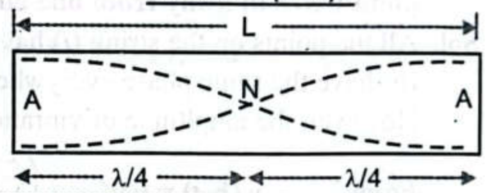
Q. 16. A steel rod 100 cm long is clamped at its middle. The fundamental frequency of longitudinal vibrations of the rod is given to be 2·53 k Hz. What is the speed of sound in steel ?

Sol. Here, $L = 100 \text{ cm} = 1 \text{ m}$, $\nu = 2{\cdot}53 \text{ k Hz} = 2{\cdot}53 \times 10^3 \text{ Hz}$

When the rod is clamped at the middle, then in the fundamental mode of vibration of the rod, a node is formed at the middle and antinode is formed at each end.

Therefore, as is clear from Fig. 10(NCT).9,

FIGURE 10(NCT).9



$$L = \frac{\lambda}{4} + \frac{\lambda}{4} = \frac{\lambda}{2}$$

$$\lambda = 2L = 2 \text{ m}$$

As, $v = \nu\lambda$

$\therefore$ $v = 2{\cdot}53 \times 10^3 \times 2 = \mathbf{5{\cdot}06 \times 10^3 \text{ ms}^{-1}}$

Q. 17. A pipe 20 cm long is closed at one end. Which harmonic mode of the pipe is resonantly excited by a 430 Hz source ? Will this source be in resonance with the pipe, if both ends are open. Speed of sound = 340 m/s.

Sol. Here, $L = 20 \text{ cm} = 0{\cdot}2 \text{ m}$, $\nu_n = 430 \text{ Hz}$, $v = 340 \text{ ms}^{-1}$

The frequency of nth normal mode of vibration of closed pipe is

$$\nu_n = (2n-1)\frac{v}{4L} \quad \therefore \quad 430 = (2n-1)\frac{340}{4 \times 0{\cdot}2}$$

$$2n - 1 = \frac{430 \times 4 \times 0{\cdot}2}{340} = 1{\cdot}02$$

$$2n = 2{\cdot}02, \quad n = 1{\cdot}01$$

Hence it will be the 1st normal mode of vibration.

In a pipe, open at both ends, we have $\nu_n = n \times \frac{v}{2L} = \frac{n \times 340}{2 \times 0{\cdot}2} = 430 \quad \therefore \quad n = \frac{430 \times 2 \times 0{\cdot}2}{340} = \mathbf{0{\cdot}5}$

As n has to be an integer, therefore, **open organ pipe cannot be in resonance with the source.**

Q. 18. Two sitar strings *A* and *B* playing the note '*Ga*' are slightly out of tune and produce beats of frequency 6 Hz. The tension in the string *A* is slightly reduced and the beat frequency is found to reduce to 3 Hz. If the original frequency of *A* is 324 Hz, what is the frequency of *B* ?

Sol. Let original freq. of sitar string A be n_a and original freq. of sitar string B be n_b

As number of beats/sec. = 6 $\therefore$ $n_b = n_a \pm 6 = 324 \pm 6 = 330$ or 318 Hz

When tension in A is reduced, its frequency reduces ($\because n \propto \sqrt{T}$)

As number of beats/sec decreases to 3 therefore, frequency of $B = 324 - 6 = \mathbf{318 \text{ Hz}}$

Q. 19. Explain why (or how) : (a) In a sound wave, a displacement node is a pressure antinode and vice-versa,
(b) Bats can ascertain distances, directions, nature and size of obstacles without any eyes,
(c) a violin note and sitar note may have the same frequency, yet we can distinguish between the two notes,
(d) Solids can support both longitudinal and transverse waves, but only longitudinal waves can propagate in gases,
(e) The shape of pulse gets distorted during propagation in a dispersive medium.