NEET Xtract

1. (a) : Let M = mass of the boat, m = mass of the stones
For floating condition, weight = upthrust
$(M + m)g = V_D \rho_w g$; where V_D = volume of water displaced

$$V_D = \frac{M}{\rho_w} + \frac{m}{\rho_w} \quad \text{...(i)}$$

After the stones are unloaded into the water,

$V_{D_1} = \frac{M}{\rho_w}$ (V_{D_1} = volume of water displaced by boat)

$V_{D_2} = \frac{m}{\rho_s}$ (V_{D_2} = volume of water displaced by stones)

$\therefore$ total volume of water displaced

$$V_{D'} = V_{D_1} + V_{D_2} = \frac{M}{\rho_w} + \frac{m}{\rho_s} \quad \text{...(ii)}$$

$\therefore \frac{m}{\rho_w} > \frac{m}{\rho_s} \Rightarrow V_D > V_{D'}$ so the water level will fall.

2. (a) : When body is half immersed, then upthrust = weight of sphere

$\Rightarrow \frac{V}{2} \times \rho_{liq} \times g = V \times \rho \times g \quad \therefore \rho = \frac{\rho_{liq}}{2}$

When body is fully immersed, then upthrust = weight of sphere + weight of water poured in sphere

$\Rightarrow V \times \rho_{liq} \times g = V \times \rho \times g + V' \times \rho_{liq} \times g \Rightarrow V' = \frac{V}{2}$

3. (a) : When the elevator move upwards with acceleration a, net acceleration = $g + a$

$\therefore$ Pressure = $h\rho(g + a)$ dyne /cm^2

$$= \frac{76 \times 13.6 \times (g + a)}{76 \times 13.6 \times g} \text{ cm of Hg } = 1 + \frac{a}{g} > 1 \text{ cm of Hg}$$

$\therefore$ Final pressure should be more than 76 cm of Hg.

4. (b) : $P = \lim_{\Delta S \to 0} \frac{F}{\Delta S}$

This equation gives up the pressure at an point whether we are moving upward, downward, side, beside, etc., the equation will not change. This will remain the same always.
$P_1 - P_2 = \rho g z$
In this equation g is written, if the elevator is accelerating upward then acceleration of elevator should also be considered and acceleration should be '$a + g$'.
Thus this equation is not be valid and will change with the acceleration of elevator.

5. (d) : The pressure at these two points are equal to atmospheric pressure hence their difference is zero.

6. (b) : As long as $\rho \le \rho_w$, pressure at the bottom of the pan would be the same everywhere, according to the Pascal's law.

7. (c) : Area of the plate = 1/2 × base × altitude
= 1/2 × 3 × 3 = 4.5 m^2
Density of oil = 0.8 × 1000 = 800 kg m^{-3}
Depth of centre of gravity of the plate from the free surface = (1/3) 3 = 1 m.
Now, total thrust on the plate =
(plate area) × (pressure at the centre of gravity of the plate)
= (4.5) (800 × 10 × 1) = 36 kN

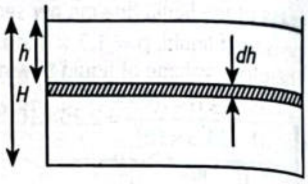
8. (a) : Force on the side walls is
F_1 = Average pressure × curved surface area

or $F_1 = \left(\frac{\rho g h}{2}\right)(2\pi R h) = \pi R \rho g h^2$ (ρ = density of liquid)

Force on the bottom is F_2 = Pressure at the bottom × area of the bottom
$= (\rho g h)(\pi R^2) = \pi \rho g h R^2$

Given, $F_1 = F_2 \quad \therefore \pi R \rho g h^2 = \pi \rho g h R^2 \Rightarrow h = R$

9. (b) : Consider an element of liquid of thickness dh as shown in the figure. Density at this location is $\rho = kh$ (where, k is a constant). Pressure due to this element is

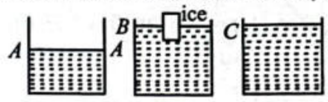


$dp = (kh)g\,dh \Rightarrow p = kg\int_0^h h\,dh$

$= \frac{1}{2}kg\,h^2 = \left(\frac{kg}{2}\right)h^2$

$p = \frac{kg}{2}h^2$ is similar to $y = kx^2$ (a parabola).

10. (a) : Let A be the initial level of water is the beaker, as shown is figure.

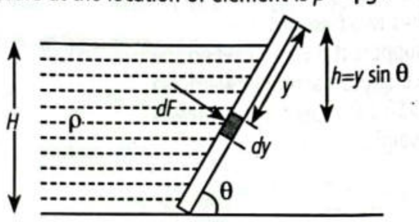


Suppose a piece of ice of mass m floats in water. The increase in the level of liquid is AB. If V be the volume of water displaced, then weight of ice = weight of water displaced.

$$mg = V\rho_w g \quad \text{or,} \quad V = \frac{m}{\rho_w}$$

when ice melts completely, let the level of water be at C. The difference of level A and C is due to the ice converted into water. If this volume be V', then $V' = \frac{m}{\rho_w}$. As $V = V'$, hence level of water remains unchanged.

11. (a) : Pressure at the location of element is $p = \rho g h$



or $p = \rho g y \sin\theta$
Force on the element is $dF = p(b dy)$,
where b is the width of the wall.
$dF = \rho g y \sin\theta\,(b dy) = \rho g b \sin\theta\, y dy$

$\therefore$ Total force on the wall due to the liquid is

$F = \rho g b \sin\theta \int_0^{H/\sin\theta} y dy$

or $F = \rho g\, b \sin\theta \frac{H^2}{2\sin^2\theta} = \frac{\rho g b H^2}{2\sin\theta}$ or $\frac{F}{b} = \frac{\rho g H^2}{2\sin\theta}$

12. (b) : Let mass of copper = m
Then, mass of gold = $50 - m$

Volume of copper is $V_c = \frac{m}{10}$

Volume of gold is $V_g = \frac{50 - m}{20}$ $\left(\because V = \frac{m}{\rho}\right)$

When immersed in water, decrease in weight = upthrust

$\therefore (50 - 46)\,g = (V_c + V_g)\,\rho_w g$

$4 = \left[\frac{m}{10} + \frac{50 - m}{20}\right](1)$ $(\because \rho_w = 1 \text{ g cc}^{-1})$

So, $80 = 2m + 50 - m \Rightarrow m = 30$ g

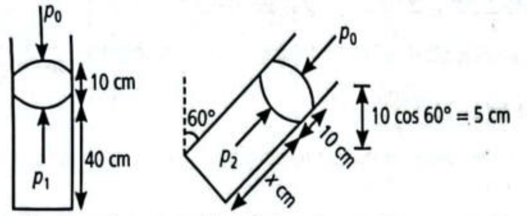
13. (b) : Upthrust in liquid A, $U_1 = \frac{1}{4}V\rho_1 g$

Upthrust in liquid B, $U_2 = \frac{3}{4}V\rho_2 g \quad \therefore \frac{U_1}{U_2} = \frac{\rho_1}{3\rho_2}$

For floatation, $\frac{1}{4}V\rho_1 g + \frac{3}{4}V\rho_2 g = V\rho g$

$\Rightarrow \rho_1 + 3\rho_2 = 4\rho$

14. (c) : $p_1 = p_0 + 10$ (in cm of Hg) and $p_2 = p_0 + 5$ (in cm of Hg)

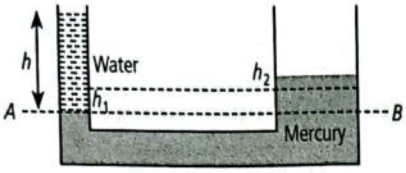


Let A be the area of cross-section of the tube. In this process, there is no temperature change. So, $p_1V_1 = p_2V_2$

$\therefore \quad (p_0 + 10)40\,A = (p_0 + 5)xA \Rightarrow (76 + 10)40 = (76 + 5)x$

$x = \frac{86}{81} \times 40 = 42.47$ cm

15. (b) : If the level in narrow tube goes down by h_1, then in wider tube level goes up to h_2



Now, $\pi r^2 h_1 = \pi(nr)^2 h_2$

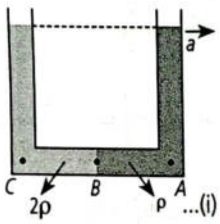
$\Rightarrow \quad h_1 = n^2 h_2$

Now pressure at point A = pressure at point B

$h\rho g = (h_1 + h_2)\,\rho' g$

$\Rightarrow \; h = (n^2h_2 + h_2)s \quad \left(\text{As } s = \frac{\rho'}{\rho}\right); \; h_2 = \frac{h}{(n^2+1)s}$

16. (b) : For the given situation, liquid of density 2ρ should be behind that of ρ.



$P_A = P_{atm} + \rho gh$

$P_B = P_A + \rho a\frac{l}{2} = P_{atm} + \rho gh + \rho a\frac{l}{2}$

$P_C = P_B + (2\rho)a\frac{l}{2} = P_{atm} + \rho gh + \frac{3}{2}\rho al$ ...(i)

But from left limb $P_c = P_{atm} + (2\rho)gh$ ...(ii)

From (i) and (ii);

$P_{atm} + \rho gh + \frac{3}{2}\rho al = P_{atm} + 2\rho gh \;\Rightarrow\; h = \frac{3a}{2g}l$

17. (d) : When the system is at rest,

B_F = Buoyant force

$mg + T_0 = B_F$

$\Rightarrow \quad V\rho_s g + T_0 = V\rho_L g$

$T_0 = Vg(\rho_L - \rho_s)$ ...(i)

When the system is accelerating, (from non-inertial frame)

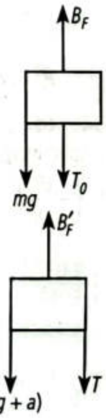
$B'_F = m(g + a) + T$

$V\rho_L(g + a) = V\rho_s(g + a) + T$

$T = V(g + a)(\rho_L - \rho_s)$ ...(ii)

On dividing Eq. (ii) by Eq. (i), we get

$\frac{T}{T_0} = \frac{g+a}{g} \;\Rightarrow\; T = T_0\left(1+\frac{a}{g}\right)$



18. (a) : The level of water does not change. The reason is that on drinking the water (say m gm), the weight of man increases by m gm and hence water displaced by man increases by m gm, tending to raise the level. However, this much amount of water has already been consumed by the man. Therefore the level of pond remain same.

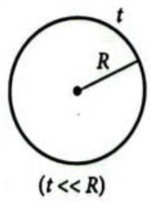
19. (b) : Here, density of metal = ρ

Mass of hollow sphere

= (Surface area) × (thickness) × $\rho = 4\pi R^2 t\rho$

Sphere will float in water if

$(4\pi R^2 t\rho)g \le \left(\frac{4}{3}\pi R^3\right)\rho_w g$



($t << R$)

or $\quad t \le \frac{R}{3}\frac{\rho_w}{\rho} \quad \because \rho_w = 1$ g cm^{-3} $\therefore \; t \le \frac{R}{3\rho}$

20. (d) : If A is the area of cross-section of a pipe at a point and v is the velocity of flow of water at that point, then by the principle of continuity

$Av = \text{constant} \Rightarrow A_1v_1 = A_2v_2$

$\Rightarrow \quad \pi r_1^2 v_1 = \pi r_2^2 v_2$

$\Rightarrow v_2 = \left(\frac{r_1}{r_2}\right)^2 v_1 = \left(\frac{1.5\times10^{-2}}{3\times10^{-2}}\right)^2 \times 4 = 1\text{ m/s}$

From Bernoulli's theorem :

$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 \Rightarrow P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

$\therefore \; P_2 = 2\times10^4 + \frac{1}{2}\times(10^3)\times(16-1) = 2\times10^4 + 7.5\times10^3$

$= 2.75 \times 10^4$ N/m^2

21. (b) : Here, $v_1 = 180$ km h^{-1} = 50 m s^{-1},

$v_2 = 234$ km h^{-1} = 65 m s^{-1}

area of the each wing, $A = 25$ m^2 and density of air, $\rho = 1$ kg m^{-3}

According to Bernoulli's theorem,

$P_1 - P_2 = \frac{1}{2}\rho(v_2^2 - v_1^2) = \frac{1}{2}(65^2 - 50^2) = 862.5$ N m^{-2}

Now, upward lift from both the wings,

$F = (P_1 - P_2) \times 2A = 862.5 \times 2 \times 25$ N

Let M be the mass of the plane. As the plane is in level flight, $F = Mg$

or $\quad M = \frac{F}{g} = \frac{862.5\times2\times25}{9.8} = 4.4\times10^3$ kg.

22. (a) : $d_p = 2 \times 10^{-2}$ m $\Rightarrow r_p = 10^{-2}$ m

$d_q = 4 \times 10^{-2}$ m $\Rightarrow r_q = 2 \times 10^{-2}$ m

Also, $a_p = \pi(r_p)^2$ and $a_q = \pi(r_q)^2$

$\frac{v_p}{v_q} = \frac{a_q}{a_p}; \; \frac{v_p}{v_q} = \frac{\pi(r_q)^2}{\pi(r_p)^2}; \; \frac{v_p}{v_q} = \frac{(2\times10^{-2})^2}{(10^{-2})^2}$

$\frac{v_p}{v_q} = 4 \Rightarrow$ Hence, $v_p = 4 \times v_q$

23. (c) : Equation of continuity is $v_1A_1 = v_2A_2$

$v_1 = 1$ m/s, $A_1 = 10^{-4}$ m^2,

v_2 = velocity of water stream at 0.15 m below the top

$A_2 = ?$

$v_2^2 - v_1^2 = 2as$; $v_2^2 - 1 = 2 \times 10 \times 0.15 \Rightarrow v_2 = 2$ m/s

Hence, $A_2 = \frac{v_1A_1}{v_2} = \frac{1\times10^{-4}}{2} = 5\times10^{-5}$ m^2

24. (d) : At point A, radius is $2r$ and at point B, radius is r.

Area × velocity = constant

Hence $\pi r^2 \times v$ = constant

i.e., $r_1^2 v_1 = r_2^2 v_2$

Given, $r_1 = 2r$; $r_2 = r$, $v_1 = v$, $v_2 = ?$

$\therefore \quad (2r)^2 \times v = r^2 \times v_2 \Rightarrow v_2 = 4v \Rightarrow$ i.e., $v_B = 4v$

25. (d) : Bernoulli's theorem for unit mass of liquid

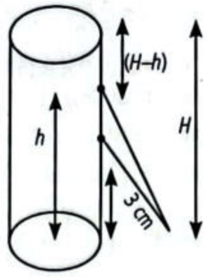
$\frac{p}{\rho} + \frac{1}{2}v^2 = \text{constant}$

As the liquid starts flowing, its pressure energy decreases.

$\frac{1}{2}v^2 = \frac{P_1 - P_2}{\rho} \Rightarrow \frac{1}{2}v^2 = \frac{5\times10^5 - 3\times10^5}{10^3}$

$\Rightarrow v^2 = \frac{2\times2\times10^5}{10^3} \quad \therefore v^2 = 400 \Rightarrow v = 20$ m/s

26. (d) : $v = \sqrt{2gh}$; $v_1 = \sqrt{2g(H-3)}$



$t_1 = \sqrt{\frac{2\times3}{g}} = \sqrt{\frac{6}{g}}$

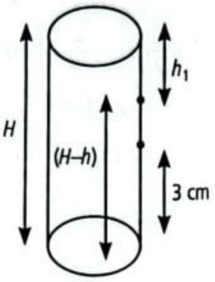
$x = v_1t_1 = \sqrt{2g(H-3)\times\frac{6}{g}} = \sqrt{12(H-3)}$

$v_2 = \sqrt{2g(H-h)}$, $t_2 = \sqrt{\frac{2h}{g}}$

$x = v_2t_2 \Rightarrow \sqrt{4h(H-h)}$

$12(H-3) = 4h(H-h)$

$3(H-3) = h(H-h)$ ...(i)



$x = \sqrt{12(H-3)}$; $t_2' = \sqrt{\frac{2(H-h_1)}{g}}$

$v_2' = \sqrt{2gh_1}$

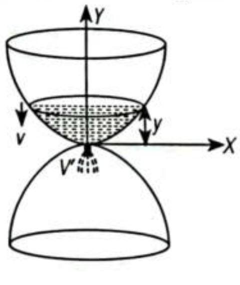
$x = \sqrt{4h_1(H-h_1)} \Rightarrow 12(H-3) = 4h_1(H-h_1)$

$\Rightarrow$ $3(H-3) = h_1(H-h_1)$ ...(ii)

On solving eqn. (i) and (ii), we get, $h = h_1 = 3$ cm

27. (d) : Since, the tubes A and C are connected to a tube of same area of cross-section, and the liquid flowing there will have same velocity, hence, the height of liquid in A and C will be same. Since, tube B is connected to a tube of smaller area of cross-section, therefore the liquid is flowing faster in this tube and pressure there is less according to Bernoulli's theorem.

28. (d) : A snapshot of the system at an instant is shown in the figure.



v = speed with which the water level lowers (this has to be constant).

$A = \pi x^2$ = area of surface of water

$v' = \sqrt{2gy}$ = speed of water through orifice.

a = area of orifice (given)

From equation of continuity, at this instant

$Av = av'$

$\pi x^2 v = a\sqrt{2gy}$

$2gy = \frac{\pi^2x^4v^2}{a^2} \Rightarrow y = \frac{\pi^2v^2}{2ga^2}\cdot x^4$

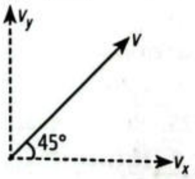
If v has to be constant, $\frac{\pi^2v^2}{2ga^2}$ = constant = k (say)

then equation required is $y = kx^4$

where, $k = \frac{\pi^2v^2}{2ga^2}$

29. (b) : Mass of water coming out in unit time is $m = (\rho A)v$

$m = (1000)\frac{\pi(0.05)^2}{4}(5) = 9.82 \text{ kg s}^{-1}$



$v_x = v\cos 45° = 5\left(\frac{1}{\sqrt{2}}\right) = 3.54 \text{ m s}^{-1}$

Applying momentum equation in x-direction

Spring force magnitude $= \frac{mv_x}{1\text{ s}} = \frac{(9.82)(3.54)}{1}$

$F_{SF} = 34.76$ N $\therefore$ $kx = 34.76$ N $\Rightarrow x = \frac{34.76}{20} = 1.74$ cm

30. (a) : Volume of water in the vessel of base area A' and height h is $V = A'h$. Average velocity of out flowing water when height of water changes from h to 0 is

$v = \frac{\sqrt{2gh}+0}{2} = \frac{\sqrt{2gh}}{2}$ $\therefore$ $V = Avt$

When vessel is filled to height $4h$, then volume in vessel

$4V = 4Avt = 4A\frac{\sqrt{2gh}}{2}\times t$

If t is the time taken for the out flowing liquid and v_1 is the average velocity of out flowing liquid, then

$4V = Av_1t_1$ or $t_1 = \frac{4V}{Av_1} = \frac{4A\sqrt{2gh}\times t\times 2}{2\times A\times\sqrt{2g\times 4h}} = 2t$

31. (a) : As, $v_1 = \frac{V}{A_1} = \frac{12\times10^{-6}}{6\times10^{-6}} = 2\text{ms}^{-1} = 200\text{ cms}^{-1}$

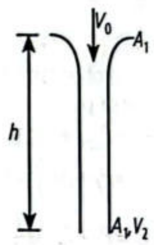
and $v_2 = \frac{V}{A_2} = \frac{12\times10^{-6}}{3\times10^{-6}} = 4\text{ms}^{-1} = 400\text{cms}^{-1}$

Now, $p_A - p_B = \rho g(h_2 - h_1) + \frac{\rho}{2}(v_2^2 - v_1^2)$

$= 1\times1000(100) + \frac{1}{2}(16\times10^4 - 4\times10^4)$

$= 10^5 + 6\times10^4 = 1.6\times10^5$ dyne cm^{-2}

32. (b) : $V_2^2 = V_0^2 + 2gh$



And $A_1V_0 = A_2V_2$

Solving, $\frac{A_2}{A_1} = \frac{V_0}{\sqrt{V_0^2+2gh}}$

$A_2/A_1 = \frac{1}{2} = \frac{V_0}{\sqrt{V_0^2+2gh}}$

$4V_0^2 = V_0^2 + 2gh$; $h = \frac{3V_0^2}{2g}$

33. (c) : From Torricelli's theorem $v = \sqrt{(2gd)}$

where v is horizontal velocity and d is the depth of water in barrel. Time t to hit the ground is given by

$h = \frac{1}{2}gt^2$ or $t = \sqrt{\frac{2h}{g}}$ $\therefore$ $R = vt = \sqrt{(2gd)}\sqrt{\frac{2h}{g}} = 2\sqrt{dh}$

$\therefore$ $R^2 = 4dh$ or $d = \frac{R^2}{4h}$

34. (d) : Let A and a be cross-sectional areas of the tank and hole respectively. Let h be height of water in the tank at time t.

If $\left(-\frac{dh}{dt}\right)$ represent the rate of fall of level, then

$-A\frac{dh}{dt} = a\sqrt{2gh}$ or $dt = -\frac{A}{a\sqrt{2g}}\frac{dh}{\sqrt{h}}$

$\therefore$ Required ratio $\frac{t_1}{t_2} = \frac{\int_h^{3h/4}\frac{dh}{\sqrt{h}}}{\int_{3h/4}^{0}\frac{dh}{\sqrt{h}}} = \frac{\left[\sqrt{\frac{3h}{4}} - \sqrt{h}\right]}{\left[0 - \sqrt{\frac{3h}{4}}\right]} = \frac{2-\sqrt{3}}{\sqrt{3}}$

35. (c) : According to equation of continuity $A_1v_1 = A_2v_2$

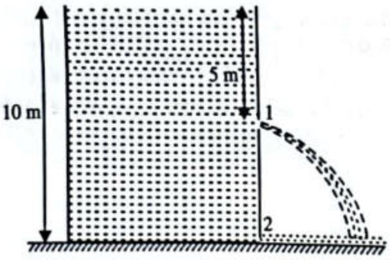
$\therefore$ $\frac{v_2}{v_1} = \frac{A_1}{A_2} = \frac{\pi r_1^2}{\pi r_2^2} = \left(\frac{r_1}{r_2}\right)^2$

or $v_2 = v_1\left(\frac{r_1}{r_2}\right)^2 = 8\left(\frac{1}{2}\right)^2 = \frac{8}{4} = 2 \text{ m s}^{-1}$

From Bernoulli's theorem, $P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2)$

$= 1.0\times10^5 + \frac{1}{2}\times10^3\times(8^2 - 2^2) = 1.3\times10^5 \text{ N m}^{-2}$

36. **(d) :**



Velocity with which water coming out from orifice 1, $v_1 = \sqrt{2g \times 5}$

Velocity with which water coming out from orifice 2, $v_2 = \sqrt{2g \times 10}$

$\therefore \quad \frac{v_1}{v_2} = \sqrt{\frac{2g \times 5}{2g \times 10}} = \frac{1}{\sqrt{2}}$

37. **(d) :** $v_1 = \sqrt{2g\left(\frac{h}{2}\right)} = \sqrt{gh}$

Using Bernoulli's theorem, we get

$P_a + \rho gh + 2\rho g\left(\frac{h}{2}\right) = P_a + \frac{1}{2}(2\rho)v_2^2 \quad \text{or} \quad v_2 = \sqrt{2gh}$

$\therefore \quad \frac{v_1}{v_2} = \frac{1}{\sqrt{2}}$

38. **(c) :** As, $p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$ (from Bernoulli's equation)

or $\quad p_1 - p_2 = \frac{1}{2}\rho(v_2^2 - v_1^2)$

$= \frac{1}{2} \times 1.3 \times (120^2 - 90^2) = 4.095 \times 10^3 \text{ Nm}^{-2}$

Gross lift on the wing $= (p_1 - p_2) \times$ area $= 4.095 \times 10^3 \times 10 \times 2$ $= 81.9 \times 10^3$ N

39. **(c) :** As, $Av = 2Av'$ or $v' = v/2$

For a horizontal pipe, according to Bernoulli's theorem

$p + \frac{1}{2}\rho v^2 = p' + \frac{1}{2}\rho\left(\frac{v}{2}\right)^2$ or $p' = p + \frac{1}{2}\rho v^2\left(1 - \frac{1}{4}\right) \Rightarrow p' = p + \frac{3}{8}\rho v^2$

40. **(d) :** Let the lower and upper surface of the wings of the aeroplane be the same height h and speeds of air on the upper and lower surfaces of the wings be v_1 and v_2.

Speed of air on the upper surface of the wing $v_1 = 70$ m/s

Speed of air on the lower surface of the wings $v_2 = 63$ m/s

Density of the air $\rho = 1.3$ kg/m^3

Area $A = 2.5$ m^2

According to Bernoulli's theorem, $p_1 + \frac{1}{2}\rho v_1^2 + \rho gh = p_2 + \frac{1}{2}\rho v_2^2 + \rho gh$

or $\; p_2 - p_1 = \frac{1}{2}\rho(v_1^2 - v_2^2)$ $\therefore$ Lifting force acting on the wings,

$F = (p_2 - p_1) \times A = \frac{1}{2}\rho(v_1^2 - v_2^2) \times A \qquad \left[\because \text{ Pressure} = \frac{\text{Force}}{\text{Area}}\right]$

$= \frac{1}{2} \times 1.3 \times [(70)^2 - (63)^2] \times 2.5$

$= \frac{1}{2} \times 1.3[4900 - 3969] \times 2.5 = \frac{1}{2} \times 1.3 \times 931 \times 2.5 = 1.51 \times 10^3$ N

41. **(a) :** Consider any three sections (1), (2) and (3) in the three pipes of different radii, as shown the figure. If v_1 and v_2 be the velocity of water at section (1) and (2) respectively, then

$a_1 = \pi(10)^2$ cm^2 ; $a_2 = \pi(5)^2$ cm^2

$a_3 = \pi(3)^2$ cm^2

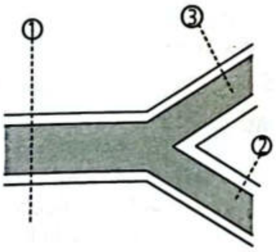
$v_1 = v_2 = ?$ and $v_3 = 5$ cm/s

The rate of discharge of the three pipes are

$a_1v_1 = 100\,\pi v_1$ cm^3/s

$a_2v_2 = 25\,\pi v_2$ cm^3/s

and $\; a_3v_3 = 9\pi \times 5 = 45\,\pi$ cm^3/s



Now, $Q = a_1v_1 = a_2v_2 + a_3v_3$

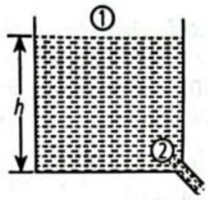
$\Rightarrow \quad 600\,\pi = 100\,\pi v_1 = 25\,\pi v_2 + 45\,\pi$

Solving, we get, $v_1 = 6$ cm/s and $v_2 = 22.2$ cm/s

42. **(d) :** The velocity of fluid at the hole is $V_2 = \sqrt{\frac{2gh}{1 + (a^2/A^2)}}$

Using the continuity equation at the two cross-section (1) and (2)

$V_1A = V_2a \Rightarrow V_1 = \frac{a}{A}V_2$



$\Rightarrow \quad$ Acceleration of the top surface $= -V_1\frac{dV_1}{dh}$

$= -\frac{a}{A}V_2\frac{d}{dh}\left(\frac{a}{A}V_2\right)$

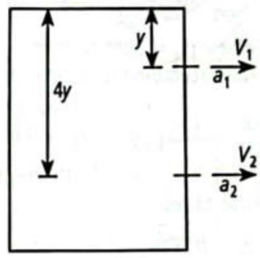
$a_1 = -\frac{a^2}{A^2}V_2\frac{dV_2}{dh} = -\frac{a^2}{A^2}\sqrt{2gh}\,\sqrt{2g}\cdot\frac{1}{2\sqrt{h}}$

$\Rightarrow \quad a_1 = \frac{-ga^2}{A^2}$

43. **(a) :** Velocity of efflux at a depth h is given by

$v = \sqrt{2gh}$

Volume of water flowing out per second from both the holes are equal.



$\therefore \quad a_1v_1 = a_2v_2$

or $\quad L^2\sqrt{2gy} = \pi R^2\sqrt{2g(4y)}$

or $\quad R = \frac{L}{\sqrt{2\pi}}$

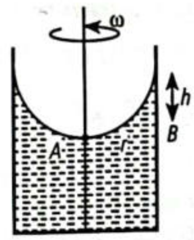
44. **(b) :** From Bernoulli's theorem,

$P_A + \frac{1}{2}dv_A^2 + dgh_A = P_B + \frac{1}{2}dv_B^2 + dgh_B$

Here, $h_A = h_B$

$\therefore \quad P_A + \frac{1}{2}dv_A^2 = P_B + \frac{1}{2}dv_B^2$

$\Rightarrow \quad P_A - P_B = \frac{1}{2}d\,[v_B^2 - v_A^2]$



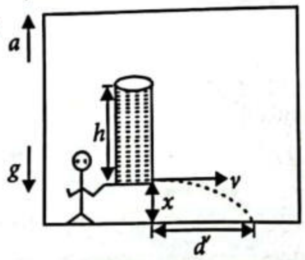
Now, $v_A = 0$, $v_B = r\omega$ and $P_A - P_B = hdg$

$\therefore \; hdg = \frac{1}{2}dr^2\omega^2$ or $h = \frac{r^2\omega^2}{2g}$

45. **(a) :** Due to Bernoulli's principle, the increase in speed of the wind between the boats causes a decrease in pressure there.

46. **(d) :** Streamline motion of a liquid is an orderly type of motion in which the liquid flow in parallel layers and every particle of liquid follows the path of its preceding particle with exactly the same velocity in magnitude and direction. In streamlined motion, the liquid must flow with velocity less than the critical velocity of the liquid. The moment at which the actual velocity of flow of liquid exceeds critical velocity, the flow becomes turbulent. For liquid to remain in steamlined motion, the limiting value of critical velocity should be as large as possible.

47. **(c) :**



When lift is moving up with acceleration a, speed of water from jar is given by,

$v = \sqrt{2(g + a)h}$ ...(i)

Time taken by water to reach the base of the lift is given by

$x=\frac{1}{2}(g+a)t^2 \Rightarrow t=\sqrt{\frac{2x}{(g+a)}}$ ...(ii)

Horizontal distance covered by water on the floor of the lift,

$d'=vt$

$=\sqrt{2(g+a)h}\times\sqrt{\frac{2x}{(g+a)}}=\sqrt{4xh}.$

So d' is independent of acceleration of the lift

∴ (A, B, C) → (P)

(D) When lift falls freely, effective acceleration of the person in the lift holding the jar is zero, so no water leaks out of the jar

$(v=\sqrt{2(g-g)h}=0)$.

(D) → (S)

So, (A)→ (P); (B)→ (P); (C)→ (P); (D)→ (S)

48. (b) : P = Normal atmospheric pressure = 1.013×10^5 Pa

Let h be the height of the French wine column which earth's atmosphere can support.

∴ If P' be the pressure corresponding to height h of wine column,

Then, $P' = h\rho_w g$

where ρ_w = density of wine = 984 kg m^{-3}

Now according to given statement, $P' = P$ or $h\rho_w g = P$

or $h=\frac{P}{\rho_w g}=\frac{1.013\times10^5}{984\times9.8}=10.5$ m.

49. (c) : Let h be the height through which the liquid rises in the capillary tube of radius r.

∴ $h=\frac{2S\cos\theta}{r\rho g}$

Mass of the water in the first tube is

$m=\pi r^2 h\rho=\pi r^2\left(\frac{2S\cos\theta}{r\rho g}\right)\rho=\frac{\pi r 2S\cos\theta}{g}$ or $m\propto r$

∴ $\frac{m'}{m}=\frac{r'}{r}=\frac{3r}{r}=3$ or $m' = 2m = 3 \times 6$ g = 18 g

50. (c) : We know that the cloth has narrow spaces in the form of capillaries. The rise of liquid in a capillary tube is directly proportional to cosθ. If θ is small cosθ will be large. Due to which capillary rise will be more and so the detergents will penetrate more in cloth as angle of contact is small.

51. (c) : As velocity at the bottom of the river will be zero,

Velocity gradient $\frac{dv}{dy}=\frac{18\times10^3}{60\times60\times5}=1\,\text{s}^{-1}$

Shear stress $=\frac{F}{A}=\eta\frac{dv}{dy}=10^{-3}\times1=1\times10^{-3}\,\text{N/m}^2$.

52. (a) : Here, l = 2 m; r = 1 cm = 0.01 m; η = 0.83 Pa s

mass of the glycerine flowing per sec, $m = 4.0 \times 10^{-3}$ kg s^{-1}

density of glycerine, $\rho = 1.3 \times 10^3$ kg m^{-3}

Therefore, volume of glycerine flowing per second,

$V=\frac{m}{\rho}=\frac{4.0\times10^{-3}}{1.3\times10^3}=3.077\times10^{-6}\,\text{m}^3\,\text{s}^{-1}$

Now, $V=\frac{\pi p r^4}{8\eta l}$

or $p=\frac{8V\eta l}{\pi r^4}=\frac{8\times3.077\times10^{-6}\times0.83\times2}{\pi\times(0.01)^4}\approx 1300$ Pa

53. (c) : Area of plate = 2 m^2

Thickness of film = 0.25 cm = 0.25×10^{-2} m

η = 15 poise = 1.5 Pa s, Speed of plate = 10 cm/s = 10×10^{-2} m/s

$F=\eta A\left(\frac{dv}{dx}\right)=\frac{1.5\times2\times10\times10^{-2}}{0.25\times10^{-2}}$ N = 120 N

$F = 1.2 \times 10^7$ dyne

54. (b) : Velocity gradient $\frac{v}{d}=\frac{0.5}{\frac{2.5}{2}\times10^{-2}}=40$

As force on the plate due to viscosity is from upper as well as lower portion of the oil, and both are equal, using the expression for viscous force,

$F_{viscous}=\eta A\frac{v}{d}$

For two surfaces, $F_{viscous}=\eta\times2A\frac{v}{d}$

∴ 1 N = $\eta \times 2 \times 0.5 \times 40$; ∴ $\eta = 2.5 \times 10^{-2}$ N s/m^2

55. (b) : Given, R = 0.04 m, η = 0.001 Pa-s, L = 4000 m

and Q = 20 Ls^{-1} = 20×10^{-3} m^3 s^{-1}

$Q=\frac{\pi}{8}\cdot\frac{R^4}{L}\cdot\frac{p_1-p_2}{\eta}$

$\Rightarrow \Delta p=p_1-p_2=\frac{8Q\eta L}{\pi R^4}=\frac{8(20\times10^{-3})(0.001)(4000)}{(3.14)(0.04)^4}$

or $\Delta p = p_1 - p_2 = 7.96 \times 10^4$ Pa

$h=\frac{\Delta p}{\rho g}=\frac{7.96\times10^4}{13600\times9.8}=0.5972\,\text{m}=59.72\,\text{cm}$

56. (d) : $\frac{4}{3}\pi R^3=8\left(\frac{4}{3}\pi r^3\right)$

$\Rightarrow R = 2r$

Terminal velocity is given by

$v_T=\frac{2}{9}\cdot\frac{r^2}{\eta}(\rho_s-\rho_L)g$ or $v_T\propto r^2$

$\Rightarrow \frac{v_{T_2}}{v_{T_1}}=\frac{R^2}{r^2}=\frac{4r^2}{r^2}=4 \Rightarrow v_{T_2}=4(v_{T_1})=4(0.1)$

or $v_{T_2}=0.4\,\text{ms}^{-1}$

57. (c) : The sphere, as it falls from a large height, has a large velocity when it enters the viscous liquid. So, it experiences a large viscous force upwards. The velocity, then keeps on decreasing. Thus, the viscous force and the magnitude of (negative) acceleration go on decreasing non-linearly. At a particular point of time, acceleration becomes zero and velocity remains constant. So, the best curve is R.

58. (d) : When we move from the centre to circumference, the velocity of liquid goes on decreasing and finally becomes zero.

59. (c) : When the pebble is falling through the viscous oil, the viscous force is $F = 6\pi\eta r v$ where r is the radius of the pebble, v is instantaneous speed and η is the coefficient of viscosity. As the force is variable, hence acceleration is also variable so v–t graph will not be straight line. First velocity increases and then becomes constant known as terminal velocity.

60. (a) : As volume of 125 droplets = Volume of the drop

∴ $125\frac{4}{3}\pi r^3=\frac{4}{3}\pi R^3$ or $125r^3 = R^3$ or $r=\frac{R}{5}$

The surface area of the drop = $4\pi R^2$

and the surface area of 125 droplets

$=125(4\pi r^2)=125\left(4\pi\left(\frac{R}{5}\right)^2\right)=20\pi R^2$

∴ The increase in surface area = $20\pi R^2 - 4\pi R^2 = 16\pi R^2$

The change in surface energy

= Surface tension × increase in surface area

= $T \times 16\pi R^2 = 16\pi R^2 T$

61. (c) : In a freely falling elevator, g = 0.

As, $h=\frac{2T\cos\theta}{r\rho g}$, so, $h=\infty$

In such a case, the radius of the meniscus will adjust itself in such a way that there would be no overflowing of water. At this stage, the length of the water column becomes equal to the length of the tube, *i.e.*, 30 cm.

62. (d) : When thousand drops coalesce, the area of free surface of water decreases. So, the corresponding surface energy (*i.e.*, energy associated with the free surface) decreases. This decrease in surface energy is equal to the heat liberated.

Final volume = initial volume $\Rightarrow \frac{4}{3}\pi R^3 = (1000)\left(\frac{4}{3}\pi r^3\right)$

$R = 10r = (10)(10^{-7})\text{ m} = 10^{-6}\text{ m}$

Water drop has only one free surface unlike a soap bubble.

$\therefore$ Decrease in surface area $= (4\pi r^2)\,1000 - 4\pi R^2$

$= 4\pi[1000r^2 - R^2] = 4\pi\,[10^{-11} - 10^{-12}]$

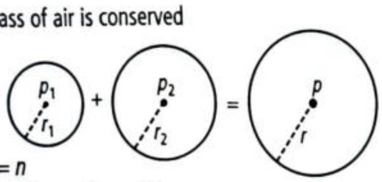
$= 4\pi \times 10^{-11}\,[1 - 0.1] = 0.9 \times 4\pi \times 10^{-11} = 1.13 \times 10^{-10}\text{ m}^2$

Surface tension (T) is numerically equal to surface energy possessed by unit surface area.

So, heat liberated = (decrease in surface area) T

$= (1.13 \times 10^{-10})(7 \times 10^{-2}) = 7.91 \times 10^{-12}\text{ J}$

63. (d) : Mass of air is conserved



$\therefore \quad n_1 + n_2 = n$

As $pV = nRT$, $\frac{p_1V_1}{RT_1} + \frac{p_2V_2}{RT_2} = \frac{pV}{RT}$

$T_1 + T_2 = T$ $\quad$ ($\because$ temperature is constant)

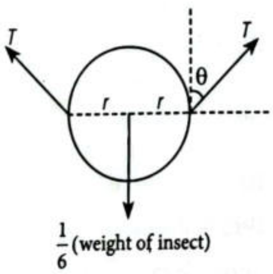
$\therefore \quad p_1V_1 + p_2V_2 = pV$

Let us denote surface tension by S. Then,

$$\left(p_0 + \frac{4S}{r_1}\right)\left(\frac{4}{3}\pi r_1^3\right) + \left(p_0 + \frac{4S}{r_2}\right)\left(\frac{4}{3}\pi r_2^3\right) = \left(p_0 + \frac{4S}{r}\right)\left(\frac{4}{3}\pi r^3\right)$$

Solving this, we get $S = \dfrac{p_0(r^3 - r_1^3 - r_2^3)}{4(r_1^2 + r_2^2 - r^2)}$

64. (a) : Insect is in equilibrium. So,



$2\pi r\,T\cos\theta = \frac{1}{6} \times$ weight of insect

$2(3.14)(2 \times 10^{-5})(0.072)\cos\theta$

$= \frac{1}{6}(0.003 \times 10^{-3} \times 9.8)$

$\therefore \quad \cos\theta \approx 0.54$

$\Rightarrow \quad \theta = \cos^{-1}(0.54)$

65. (c) : Outside pressure = 1 atm

Pressure inside first bubble = 1.01 atm

Pressure inside second bubble = 1.02 atm

Excess pressure $\Delta P_1 = 1.01 - 1 = 0.01$ atm

Excess pressure $\Delta P_2 = 1.02 - 1 = 0.02$ atm

$\Delta P \propto \frac{1}{r} \Rightarrow r \propto \frac{1}{\Delta P} \Rightarrow \frac{r_1}{r_2} = \frac{\Delta P_2}{\Delta P_1} = \frac{0.02}{0.01} = \frac{2}{1}$

Since $V = \frac{4}{3}\pi r^3 \therefore \frac{V_1}{V_2} = \left(\frac{r_1}{r_2}\right)^3 = \left(\frac{2}{1}\right)^3 = \frac{8}{1}$

66. (b) : Let n drops coalesce to form the big drop. Then

$n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $n = \frac{R^3}{r^3}$, As heat produced $= \frac{\text{work done}}{J}$

$\therefore \quad ms\Delta T = \frac{\text{surface tension} \times \text{decrease in area}}{J}$

or $\quad ms\Delta T = \frac{S}{J} \times [n \times 4\pi r^2 - 4\pi R^2]$

or $\quad \frac{4}{3}\pi R^3 \times 1 \times 1 \times \Delta T = \frac{S}{J} \times [n \times 4\pi r^2 - 4\pi R^2]$

or $\quad \Delta T = \frac{3S \times 4\pi}{J\,4\pi R^3}\,[nr^2 - R^2] = \frac{3S}{JR^3}\left[\frac{R^3}{r^3}r^2 - R^2\right]$

$= \frac{3\,SR^3}{JR^3}\left[\frac{1}{r} - \frac{1}{R}\right] = \frac{3S}{J}\left[\frac{1}{r} - \frac{1}{R}\right]$

67. (a) : Here, radius of the drop, $R = 1\text{ cm} = 1 \times 10^{-2}\text{ m}$

Let r be the radius of each droplet. Then

Volume of 10^6 droplets = Volume of the drop

or $\quad 10^6 \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3$ or $r = \frac{10^{-2}}{10^2} = 10^{-4}\text{ m}$

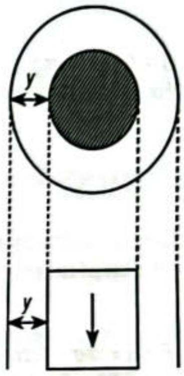
Increase in surface area $= 10^6 \times 4\pi r^2 - 4\pi R^2$

$= 10^6 \times 4 \times \pi \times 10^{-8} - 4\pi \times 10^{-4} = 4 \times 9.9 \times \pi \times 10^{-3}\text{m}^2$

$\therefore$ Work done = surface tension $\times$ increase in surface area

$= 35 \times 10^{-2} \times 4 \times 9.9 \times \pi \times 10^{-3} = 4.35 \times 10^{-2}$ J

68. (b) : Thickness of annular space



$= \frac{20.0628 - 20}{2} = 0.0314\text{ cm} = 0.000314\text{ m}$

In steady state,

Gravitational force = Viscous force

or $\quad mg = \eta A\frac{\Delta v}{\Delta y}$

But $\quad A = 2\pi rl$

$= 2 \times 3.14 \times (10 \times 10^{-2})(20 \times 10^{-2})$

$= 0.1256\text{ m}^2$

From Eq. (i)

$\therefore \quad 1 \times 10 = (10 \times 10^{-1})(0.1256)\left(\frac{v - 0}{0.000314}\right)$

$\Rightarrow \quad v = 0.025\text{ ms}^{-1} = 2.5\text{ cm s}^{-1}$

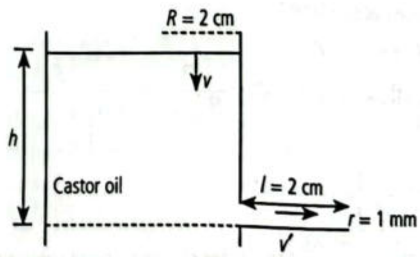
69. (b) : Statement-1 is true, Statement-2 is true but Statement-2 is not a correct explanation for Statement-1.

Assuming, the rain drops as spherical, the stoke's force on it would be $6\pi\eta rv$. As radius 'r' for raindrops is different, the terminal velocity acquired by raindrop would be different. $mg = 6\pi\eta r \cdot v_t$

70. (c) : Volume flow rate in capillary tube is

$Q = \frac{V}{t} = \frac{\pi r^4}{8\eta l}\Delta p$ $\quad$ ...(i)

$\Delta p = \rho gh$ $\quad$ ...(ii)



Also, volume flow rate in capillary tube

$\frac{V}{t} = A'v' = \pi r^2 v'$ $\quad$ ...(iii)

Using Eqs. (ii) and (iii) in Eq. (i) we get

$\pi r^2 v' = \frac{\pi r^4}{8\eta l}\cdot \rho gh$ or $v' = \frac{r^2\rho gh}{8\eta l}$ $\quad$ ...(iv)

From equation of continuity, $v'A' = vA$; $\pi r^2 v' = v\pi R^2$

$\Rightarrow v = \frac{r^2}{R^2}\cdot v'$ or $v = \frac{r^2}{R^2}\cdot\frac{r^2\rho gh}{8\eta l} = \frac{r^4\rho gh}{8R^2\eta l}$ $\quad$ [from Eq. (iv)]

71. (a) : Terminal velocity (for the given depth and time) is

$v_T = \frac{s}{t} = \frac{2 \times 10^{-2}\text{ m}}{3600\text{ s}} = 5.56 \times 10^{-6}\text{ ms}^{-1}$