Test 8

System of Particles and Rotational Motion

SCI NEET Physics

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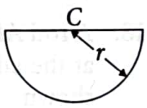
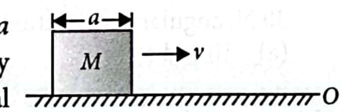
1. A semicircular lamina of mass m, radius r and centre at C is shown in the figure. Its centre of mass is at a distance x from C. Its moment of inertia about an axis through its centre of mass and perpendicular to its plane is



(a) $\frac{1}{2}mr^2$ (b) $\frac{1}{4}mr^2$

(c) $\frac{1}{2}mr^2 + mx^2$ (d) $\frac{1}{2}mr^2 - mx^2$

2. A cubical block of side a is moving with velocity v on a horizontal smooth plane as shown in figure. It hits a ridge at point O. The angular speed of the block after it hits O is



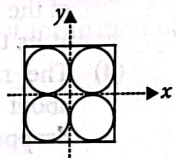
(a) $\frac{3v}{4a}$ (b) $\frac{3v}{2a}$ (c) $\frac{\sqrt{3v}}{\sqrt{2a}}$ (d) zero

3. A wheel is rotating freely at angular speed 800 rev/min on a shaft whose rotational inertia is negligible. A second wheel, initially at rest and with twice the rotational inertia of the first is suddenly coupled to the shaft. The angular speed of the resultant combination of the shaft and two wheels is

(a) 267 rev/min (b) 335 rev/min

(c) 400 rev/min (d) 1150 rev/min

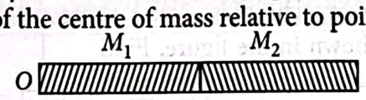
4. Four holes of radius R each are cut from a thin square plate of side $4R$ and mass M, as shown in figure. The moment of inertia of the remaining portion about z-axis is



(a) $\frac{\pi}{12}MR^2$ (b) $\left(\frac{4}{3} - \frac{\pi}{4}\right)MR^2$

(c) $\left(\frac{4}{3} - \frac{\pi}{6}\right)MR^2$ (d) $\left(\frac{8}{3} - \frac{5\pi}{8}\right)MR^2$

5. Two uniform rods of different materials M_1 and M_2 have lengths 2 m and 3 m, respectively. The mass per unit length of rods M_1 and M_2 are 1 kg/m and 2 kg/m, respectively. If the rods are arranged, as shown, the position of the centre of mass relative to point O is



(a) 4.9 m (b) 3.9 m (c) 2.9 m (d) 2.2 m

6. Three masses are placed on the x-axis : 300 g at origin, 500 g at x = 40 cm and 400 g at x = 70 cm. The distance of the centre of mass from the origin is

(a) 40 cm (b) 45 cm (c) 50 cm (d) 30 cm

7. Two particles of masses 1 kg and 2 kg are located at $x_1 = 0, y_1 = 0$ and $x_2 = 1, y_2 = 0$ respectively. The centre of mass of the system is at

(a) $x = 1, y = 2$ (b) $x = 2, y = 1$

(c) $x = \frac{1}{3}, y = \frac{2}{3}$ (d) $x = \frac{2}{3}, y = 0$

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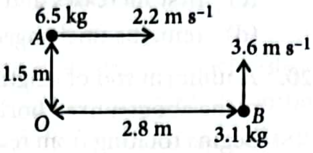
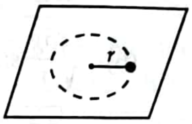
8. A child stands at one end of a boat moving with a speed v in still water. If the child starts running towards the other end of the boat with a speed u, the centre of mass of the system (boat and child) will move with a speed
(a) $v - u$ (b) v (c) u (d) $v + u$

9. Two particles A and B are moving as shown in the figure. Their total angular momentum about the point O is



(a) 9.8 kg m^2/s
(b) zero
(c) 52.7 kg m^2/s
(d) 37.9 kg m^2/s

10. A small mass attached to a string rotates on a frictionless table top as shown. If the tension in the string is increased by pulling the string causing the radius of the circular motion to decrease by a factor of 2, the kinetic energy of the mass will

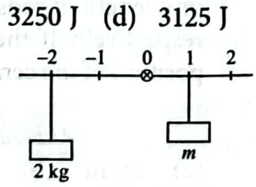


(a) decrease by a factor of 2
(b) remain constant
(c) increase by a factor of 2
(d) increase by a factor of 4

11. (1) Centre of gravity (C.G.) of a body is the point at which the weight of the body acts.
(2) Centre of mass coincides with the centre of gravity if the earth is assumed to have infinitely large radius.
(3) To evaluate the gravitational field intensity due to any body at an external point, the entire mass of the body can be considered to be concentrated at its C.G.
(4) The radius of gyration of any body rotating about an axis is the length of the perpendicular dropped from the C.G. of the body to the axis.
Which one of the following pairs of statements is correct?
(a) (4) and (1) (b) (1) and (2)
(c) (2) and (3) (d) (3) and (4)

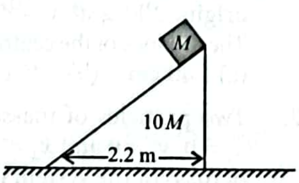
12. A solid cylinder of mass 20 kg rotates about its axis with angular speed 100 rad/s. The radius of the cylinder is 0.25 m. The kinetic energy associated with the rotation of the cylinder is
(a) 3025 J (b) 3225 J (c) 3250 J (d) 3125 J

13. A horizontal beam is pivoted at 0 as shown in the figure. Find the mass m to make the scale straight.



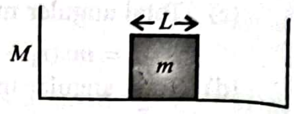
(a) 2 kg (b) 1 kg (c) 4 kg (d) 2.5 kg

14. A block of mass M is placed on top of a bigger block of mass 10 M as shown. All the surfaces are frictionless. System is released from rest. Find distance moved by bigger block at instant the smaller block reached ground.



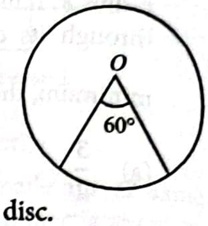
(a) 0.2 m (b) 0.5 m (c) 0.8 m (d) 0.12 m

15. Consider a gravity free hall in which a tray of mass M, carrying a cubical block of ice of mass 'm' and edge of 'L' is at rest in middle. If ice melts by what distance does the centre of mass of 'tray plus ice' system descend?



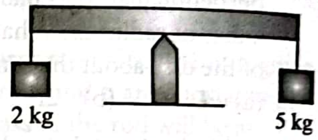
(a) ML (b) mL
(c) zero (d) $(M + m) L$

16. The uniform disc shown in figure has a moment of inertia 0.6 kg m^2 around the axis that passes through O and is perpendicular to the plane of page. If a segment is cut out from the disc as shown, what is the moment of inertia of the remaining disc.



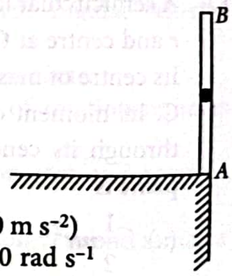
(a) 0.3 kg m^2 (b) 0.5 kg m^2
(c) 0.9 kg m^2 (d) 0.7 kg m^2

17. A rod of length 1 m is pivoted at its centre and two masses of 5 kg and 2 kg are hung from ends as shown. The rod has a mass of 1 kg distributed uniformly over its length. Find initial angular acceleration of rod assuming that it was horizontal in beginning.



(a) 4.21 rad/sec^2 (b) 8.03 rad/sec^2
(c) 12.42 rad/sec^2 (d) 15.46 rad/sec^2

18. A rod AB of length 1 m is placed at the edge of a smooth table as shown in figure. It is hit horizontally at point B. If the displacement of centre of mass in 1 s is $5\sqrt{2}$ m. The angular velocity of the rod is (Take $g = 10$ m s^{-2})



(a) 30 rad s^{-1} (b) 20 rad s^{-1}
(c) 10 rad s^{-1} (d) 5 rad s^{-1}

19. A rope is wound round a hollow cylinder of mass 3 kg and radius 40 cm. If the rope is pulled with a force of 30 N, angular acceleration of the cylinder will be
(a) 10 rad s^{-2} (b) 15 rad s^{-2}
(c) 20 rad s^{-2} (d) 25 rad s^{-2}

20. A carpet of mass M, made of an extensible material is rolled along its length in the form of a cylinder of radius R and kept on a rough floor. If the carpet is unrolled, without sliding to a radius $R/2$, the decrease in potential energy is
(a) $\frac{1}{2} MgR$ (b) $\frac{7}{8} MgR$
(c) $\frac{5}{8} MgR$ (d) $\frac{3}{4} MgR$

21. The moment of inertia of a body about a given axis is 1.2 kg m^2. Initially, the body is at rest. In order to produce a rotational kinetic energy of 6000 joule, an angular acceleration of 25 rad s^{-2} must be applied

about that axis for a duration of
(a) 4 s (b) 2 s (c) 8 s (d) 10 s

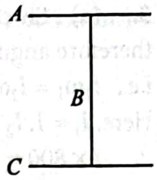
22. The maximum and minimum distances of a comet from the sun are 1.4×10^{12} m and 7×10^{10} m. If its velocity nearest to the sun is 6×10^4 m s^{-1}, what is the velocity in the farthest position? Assume that path of the comet in both the instantaneous positions is circular.
(a) 3000 m s^{-1} (b) 5000 m s^{-1}
(c) 8000 m s^{-1} (d) 12000 m s^{-1}

23. A star of mass twice the solar mass and radius 10^6 km rotates about its axis with an angular speed of 10^{-6} rad s^{-1}. What is the angular speed of the star when it collapses (due to inward gravitational force) to a radius of 10^4 km ? Solar mass 1.99×10^{30} kg.
(a) 0.03 rad s^{-1} (b) 3 rad s^{-1}
(c) 0.01 rad s^{-1} (d) 1 rad s^{-1}

24. A tennis racket can be idealized as a uniform ring of mass M and radius R, attached to a uniform rod also of mass M and length L. The rod and the ring are coplanar, and the line of the rod passes through the centre of the ring. The moment of inertia of the object (racket) about an axis through the centre of the ring and perpendicular to its plane is
(a) $\frac{1}{12}M(6R^2+L^2)$
(b) $\frac{1}{12}M(18R^2+L^2)$
(c) $\frac{1}{3}M(6R^2+L^2+3LR)$
(d) none of these

25. An air compressor is powered by a 200 rad s^{-1} electric motor using a V-belt drive. The motor pulley is 8 cm in radius, and the tension in the V-belt is 135 N on one side and 45 N on the other. The power of the motor will be
(a) 1.44 kW (b) 14.4 kW
(c) 2.88 kW (d) 28.8 kW

26. In the diagram shown below all three rods are of equal length L and equal mass M. The system is rotated such that rod B is the axis. What is the moment of inertia of the system?



(a) $\frac{ML^2}{6}$ (b) $\frac{4}{3}ML^2$ (c) $\frac{ML^2}{3}$ (d) $\frac{2}{3}ML^2$

27. If the earth were to suddenly contract to $\frac{1}{n}$th of its present radius without any change in its mass, the duration of the new day will be nearly
(a) $\frac{24}{n}$ hr (b) $24n$ hr
(c) $\frac{24}{n^2}$ hr (d) $24n^2$ hr

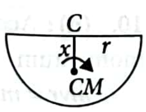
28. A solid cylinder is rolling down on an inclined plane of angle θ. The coefficient of static friction between the plane and cylinder is μ_s. The condition for the cylinder not to slip is
(a) $\tan\theta \geq 3\mu_s$ (b) $\tan\theta > 3\mu_s$
(c) $\tan\theta \leq 3\mu_s$ (d) $\tan\theta < 3\mu_s$

29. A solid cylinder and a hollow cylinder, both of the same mass and same external diameter are released from the same height at the same time on an inclined plane. Both roll down without slipping. Which one will reach the bottom first?
(a) Both together only when angle of inclination of plane is 45°
(b) Both together
(c) Hollow cylinder
(d) Solid cylinder

30. Three bodies a ring (R), a solid cylinder (C) and a solid sphere (S) having same mass and same radius roll down the inclined plane without slipping. They start from rest, if v_R, v_C and v_S are velocities of respective bodies on reaching the bottom of the plane, then
(a) $v_R = v_C = v_S$ (b) $v_R > v_C > v_S$
(c) $v_R < v_C < v_S$ (d) $v_R = v_C > v_S$

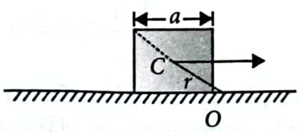
SOLUTIONS

1. **(d) :** We know, $I_C = mr^2/2$
Using parallel axes theorem,



$I_C = I_{CM} + mx^2$
$\therefore \quad I_{CM} = I_C - mx^2 = mr^2/2 - mx^2$

2. **(a) :** The block would start rotating about an axis passing through the point O.



Since no external torque acts on the block, its angular momentum is conserved. Angular momentum of the block before hitting the ridge $= mv\left(\frac{a}{2}\right)$

Angular momentum of the block after hitting the ridge $= I_o\omega$

The moment of inertia, I_o, of the block about the axis passing through the point O is

$I_o = I_C + Mr^2$

$$= \frac{Ma^2}{6} + M\left(\frac{a^2}{4}+\frac{a^2}{4}\right) = \frac{Ma^2}{6} + \frac{Ma^2}{2} = \frac{2}{3}Ma^2$$

Hence, the conservation of angular momentum gives

$$mv\left(\frac{a}{2}\right) = \left(\frac{2}{3}ma^2\right)\omega \text{ or } \omega = \frac{3v}{4a}$$

3. **(a) :** Since the external torque on system is zero, therefore angular momentum is conserved.

i.e., $I_1\omega_1 = I_2\omega_2$

Here, $I_1 = I$, $I_2 = 3I$, $\omega_1 = 800$ rev/min

$\therefore \quad I \times 800 = 3I\omega_2$

or $\quad \omega_2 = \dfrac{800}{3} \simeq 267$ rev/ min

4. **(d) :** If M is mass of the square plate before cutting the holes, then mass of each hole,

$$m = \frac{M}{16R^2} \times \pi R^2 = \frac{\pi}{16} M$$

$\therefore$ Moment of inertia of remaining portion about the given axis is

$$I = I_{\text{square}} - 4I_{\text{hole}}$$

$$= \frac{M}{12}(16R^2 + 16R^2) - 4\left[\frac{mR^2}{2} + m(\sqrt{2}R)^2\right]$$

$$= \frac{M}{12} \times 32R^2 - 10mR^2 = \frac{8}{3}MR^2 - \frac{10\pi}{16}MR^2$$

$$I = \left(\frac{8}{3} - \frac{5\pi}{8}\right)MR^2$$

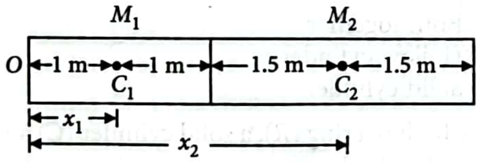
5. **(c) :** Here,

Mass per unit length of rod M_1, $\mu_1 = 1$ kg/m

Mass per unit length of rod M_2, $\mu_2 = 2$ kg/m

Length of rod M_1, $L_1 = 2$ m

Length of rod M_2, $L_2 = 3$ m



Mass of rod M_1,

$m_1 = \mu_1 L_1 = (1\text{ kg/m})(2\text{ m}) = 2$ kg (situated at C_1)

Mass of rod M_2,

$m_2 = \mu_2 L_2 = (2\text{ kg/m})(3\text{ m}) = 6$ kg (situated at C_2)

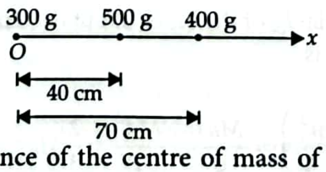
From figure, $x_1 = 1$ m and $x_2 = 3.5$ m

Thus, the position of centre of mass of the given system relative to point O is

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2}$$

$$= \frac{(2\text{ kg})(1\text{ m}) + (6\text{ kg})(3.5\text{ m})}{2\text{ kg} + 6\text{ kg}} = 2.9\text{ m}$$

6. **(a) :**



The distance of the centre of mass of the system of three masses from the origin O is

$$X_{CM} = \frac{m_1x_1 + m_2x_2 + m_3x_3}{m_1 + m_2 + m_3}$$

$$= \frac{300 \times 0 + 500 \times 40 + 400 \times 70}{300 + 500 + 400}$$

$$= \frac{500 \times 40 + 400 \times 70}{1200} = \frac{400[50 + 70]}{1200}$$

$$= \frac{50 + 70}{3} = \frac{120}{3} = 40\text{ cm}$$

7. **(d) :** Here, $m_1 = 1$ kg, $x_1 = 0$, $y_1 = 0$,

$m_2 = 2$ kg, $x_2 = 1$, $y_2 = 0$

Coordinates of the centre of mass are

$$X_{CM} = \frac{m_1x_1 + m_2x_2}{m_1 + m_2} = \frac{1 \times 0 + 2 \times 1}{3} = \frac{2}{3}$$

$$Y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{0 + 0}{3} = 0$$

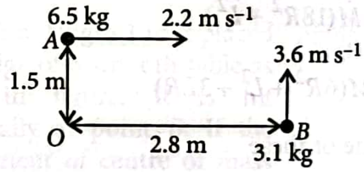
$\left(\dfrac{2}{3}, 0\right)$ is the centre of mass.

8. **(b) :** Since in the absence of any external force, the centre of mass the system does not change its velocity.

$\therefore$ The speed of centre of mass of the system in given case will be v.

9. **(a) :** Total angular momentum of two particles about the point O

$$\vec{L}_{\text{net}} = \vec{L}_A + \vec{L}_B = \vec{r}_A \times \vec{p}_A + \vec{r}_B \times \vec{p}_B$$



Here, $\vec{r}_A = 1.5\text{ m }\hat{j}$, $\vec{r}_B = 2.8\text{ m }\hat{i}$

$$\vec{p}_A = m_A\vec{v}_A = (6.5)(2.2\hat{i}) = (14.3\hat{i})\text{ kg m s}^{-1}$$

$$\vec{p}_B = m_B\vec{v}_B = (3.1)(3.6\hat{j}) = (11.16\hat{j})\text{ kg m s}^{-1}$$

$$\therefore \quad \vec{L}_{\text{net}} = (1.5\hat{j}) \times (14.3\hat{i}) + (2.8\hat{i}) \times (11.16\hat{j})$$

$$= 21.45(-\hat{k}) + 31.248(\hat{k}) \approx 9.8\hat{k}$$

or $\quad |\vec{L}_{\text{net}}| = 9.8\text{ kg m}^2\text{s}^{-1}$

10. **(d) :** According to law of conservation of angular momentum

$$mvr = mv'r'$$

or $\quad vr = v'\left(\dfrac{r}{2}\right)$ or $v' = 2v$ ...(i)

$$\therefore \quad \frac{K}{K'} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv'^2} = \left(\frac{v}{v'}\right)^2$$

or $\quad \dfrac{K'}{K} = \left(\dfrac{v'}{v}\right)^2 = (2)^2$ (Using (i))

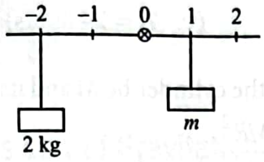
or $\quad K' = 4K$

11. (a)

12. (d) : Kinetic energy of rotation $= \frac{1}{2}I\omega^2$

$$= \frac{1}{2}\times\left(\frac{1}{2}MR^2\right)\omega^2 = \frac{1}{4}\times 20\times(0.25)^2\times 100\times 100 = 3125\text{ J}$$

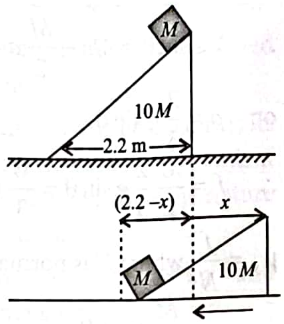
13. (c) :



Taking the moments about 0, we get

$mg(1) = 2g(2)$ or $m = 4$ kg

14. (a) : When the system is released, mass M will slide down and at the same time on the frictionless surface bigger block of mass 10 M also shifts backward. As no external force act so no shift in centre of mass. Say bigger block shifts right by 'x'. The smaller block will shift right by $(2.2 - x)$.



Shift in center of mass

$$= \frac{m_1\Delta x_1 + m_2\Delta x_2}{m_1 + m_2}$$

$$0 = \frac{[M(2.2 - x)] + 10Mx}{M + 10M}$$

$M[2.2 - x] = 10\,Mx$

$2.2 = 11x$

$x = 0.2$ m

15. (c) : Since the hall is free from gravity, hence no shift in center of mass of ice into water on melting.

16. (b) : M.I. of segment, $I' = \frac{I}{6}$

$$I' = \frac{0.6}{6} = 0.1\text{ kg m}^2$$

M.I. of remaining portion

$I = I_{Total} - I' = 0.6 - 0.1 = 0.5$ kg m^2.

17. (b) : Moment of inertia of the whole system about pivot.

$$I = 2[0.5]^2 + 5[0.5]^2 + \frac{mL^2}{12};\ I = 0.5 + 1.25 + \frac{1[1]^2}{12}$$

$I = 1.83$ kg m^2

Net torque, $\tau = 5g[0.5] - 2g[0.5]$

$\tau = 3g[0.5] = 1.47$ N m

Now, $\tau = I\alpha$

$1.47 = 1.83\,\alpha$

$$\alpha = 8.03\ \frac{\text{rad}}{\text{sec}^2}.$$

18. (a) : Let v be the velocity of rod in horizontal direction then displacement of centre of mass in horizontal direction in 1 s will be

$x = vt = v$

Displacement of centre of mass in vertical direction in 1 s will be

$$y = \frac{1}{2}gt^2 = \frac{1}{2}(10)(1)^2 = 5$$

Given that $\sqrt{x^2 + y^2} = 5\sqrt{2}$ or $\sqrt{v^2 + 25} = 5\sqrt{2}$

$\therefore\ v = 5$ m s^{-1}

Now, let J be the impulse applied at B. Then Impulse = change in momentum

or $J = mv$ ...(i)

and $J\frac{l}{2} = I\omega$ ...(ii)

Dividing (ii) by (i), we get

$$\omega = \frac{mvl}{2I} = \frac{mvl}{2\left(\frac{ml^2}{12}\right)} \text{ or } \omega = \frac{6v}{l} = \frac{6\times 5}{1} = 30\text{ rad s}^{-1}$$

19. (d) : Moment of inertia of the hollow cylinder about its axis is

$I = MR^2 = 3 \times (0.40)^2 = 0.48$ kg m^2

Torque applied on the cylinder,

τ = force × moment arm = 30 × 0.40 = 12 N m

Angular acceleration of the cylinder,

$$\alpha = \frac{\tau}{I} = \frac{12}{0.48} = 25\text{ rad s}^{-2}$$

20. (b) : The centre of mass of the whole carpet is originally at a height R above the floor. When the carpet unrolls itself and has a radius $R/2$, the centre of mass is at a height $R/2$.

The mass left over unrolled is $\frac{M\pi(R/2)^2}{\pi R^2} = \frac{M}{4}$

Decrease in potential energy

$$= MgR - \left(\frac{M}{4}\right)g\left(\frac{R}{2}\right) = \frac{7}{8}MgR$$

21. (a) : Rotational kinetic energy

$$K_R = \frac{1}{2}I\omega^2 = \frac{1}{2}I(\alpha t)^2 = \frac{1}{2}I\alpha^2 t^2$$

Substituting the given values, we get

$6000 = \frac{1}{2}\times 1.2\times(25)^2\times t^2$ or $t^2 = 16$ or $t = 4$ s

22. (a) : At minimum distance, $r_1 = 7\times 10^{10}$ m,

velocity, $v_1 = 6\times 10^4$ m s^{-1}

At maximum distance, $r_2 = 1.4\times 10^{12}$ m,

velocity, $v_2 = ?$

By conservation of angular momentum, $I_1\omega_1 = I_2\omega_2$

or $mr_1^2\times\frac{v_1}{r_1} = mr_2^2\cdot\frac{v_2}{r_2}$ or $v_1r_1 = v_2r_2$

or $$v_2 = \frac{v_1r_1}{r_2} = \frac{6\times 10^4\times 7\times 10^{10}}{1.4\times 10^{12}} = 3000\text{ m s}^{-1}.$$

23. (c) : During collapse, the total angular momentum of an isolated star is conserved, hence

$I_1\omega_1 = I_2\omega_2$

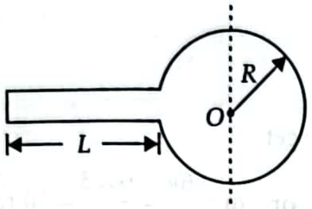
or $\frac{2}{5}MR_1^2\omega_1 = \frac{2}{5}MR_2^2\omega^2$ $\left[\because I = \frac{2}{5}MR^2\right]$

or $R_1^2\omega_1 = R_2^2\omega_2 \quad \therefore \quad \omega_2 = \frac{R_1^2}{R_2^2}\omega_1$

But $R_1 = 10^6$ km, $R_2 = 10^4$ km, $\omega_1 = 10^{-6}$ s^{-1}

$\therefore \quad \omega_2 = \frac{(10^6)^2}{(10^4)^2}\times 10^{-6} = 0.01 \text{ rad s}^{-1}$.

24. (c) :



Moment of inertia of the ring about the given axis is $I_{ring} = MR^2$

Moment of inertia of rod about the given axis is

$$I_{rod} = \frac{1}{12}ML^2 + M\left(\frac{L}{2}+R\right)^2$$

(Using theorem of parallel axes)

$$= \frac{1}{12}ML^2 + M\left(\frac{L^2}{4}+R^2+LR\right)$$

$$= \frac{1}{12}ML^2 + \frac{1}{4}ML^2 + MR^2 + MLR$$

Moment of inertia of the racket about the given axis is

$$I = I_{ring} + I_{rod}$$

$$= MR^2 + \frac{1}{12}ML^2 + \frac{1}{4}ML^2 + MR^2 + MLR$$

$$= \frac{12MR^2 + ML^2 + 3ML^2 + 12MR^2 + 12MLR}{12}$$

$$= \frac{24MR^2 + 4ML^2 + 12MLR}{12}$$

$$= 2MR^2 + \frac{1}{3}ML^2 + MLR$$

$$= \frac{1}{3}M(6R^2 + L^2 + 3LR)$$

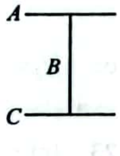
25. (a) : Since the net force on the belt is the difference between the two tensions, the torque exerted by the motor on the belt is

$\tau = Fr = (135 \text{ N} - 45 \text{ N})(0.08 \text{ m}) = 7.2 \text{ N m}$

The power of the motor is

$P = \tau\omega = (7.2 \text{ N m})(200 \text{ rad s}^{-1}) = 1440 \text{ W} = 1.44 \text{ kW}$

26. (a) : Since the width is negligible, the moment of inertia of B is zero along its length.



Moment of inertia of the system about B is

$$I = I_A + I_B + I_C = \frac{ML^2}{12} + 0 + \frac{ML^2}{12} = \frac{ML^2}{6}$$

27. (c) : According to law of conservation of angular momentum

$I_1\omega_1 = I_2\omega_2$

$$\frac{2}{5}MR^2\left(\frac{2\pi}{T_1}\right) = \frac{2}{5}M\left(\frac{R}{n}\right)^2\frac{2\pi}{T_2}$$

$T_2 = \frac{T_1}{n^2} = \frac{24}{n^2}$ hr $\quad (\because T_1 = 24 \text{ hours})$

28. (c) : Let the mass of the cylinder be M and its radius R.

Its moment of inertia $= \frac{MR^2}{2}$

For rolling without slipping the linear acceleration a for the centre of mass $= R\alpha$ where α is the angular acceleration of rotation of the cylinder.

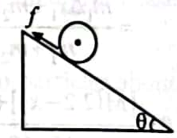
For translation, $Mg\sin\theta - f = Ma$ where f is the force of friction.

For rotation, the torque $fR = I\alpha$

But $R\alpha = a \quad \therefore \quad f = \frac{M}{2}a$

$\therefore \quad Mg\sin\theta = Ma + \frac{M}{2}a = \frac{3}{2}Ma$

or $\quad a = \frac{2}{3}g\sin\theta$



$\therefore \quad f = \frac{M}{2}\frac{2}{3}g\sin\theta = \frac{M}{3}g\sin\theta$

$\mu_s = \frac{f}{N}$ where N is normal reaction, $Mg\cos\theta$

$$\therefore \quad \mu_s = \frac{\frac{M}{3}g\sin\theta}{mg\cos\theta} = \frac{\tan\theta}{3}$$

$\therefore$ For rolling without slipping

$\mu_s \geq \frac{1}{3}\tan\theta$ or $\tan\theta \leq 3\mu_s$

29. (d) : Time taken to reach the bottom of inclined plane is

$$t = \sqrt{\frac{2l\left(1+\frac{k^2}{R^2}\right)}{g\sin\theta}}$$

Here, l is the length of incline plane.

For solid cylinder, $k^2 = \frac{R^2}{2}$

For hollow cylinder, $k^2 = R^2$

Hence, solid cylinder will reach the bottom first.

30. (c) : Velocity of the body at the bottom of incline when it rolls down the inclined plane,

$$v = \sqrt{\frac{2gh}{(1+I/mR^2)}}$$

$I_R = mR^2$, $I_C = \frac{1}{2}mR^2$, $I_S = \frac{2}{5}mR^2$

$\because \quad I_R > I_C > I_S \quad \therefore \quad v_R < v_C < v_S$

OOO