6(b).10. REFRACTION FROM RARER TO DENSER MEDIUM AT A CONVEX SPHERICAL REFRACTING SURFACE

Two cases arise. The image formed may be real or virtual.

(a) Real image

Let a spherical refracting surface XY separate a rarer medium of refractive index μ_1 from a denser medium of refractive index μ_2. Suppose the surface is convex towards rarer medium side. Let P be the pole, C be the centre of curvature and $R = PC$ be the radius of curvature of this surface.

Consider a point object O lying on the principal axis of the surface, Fig. 6(b).22.

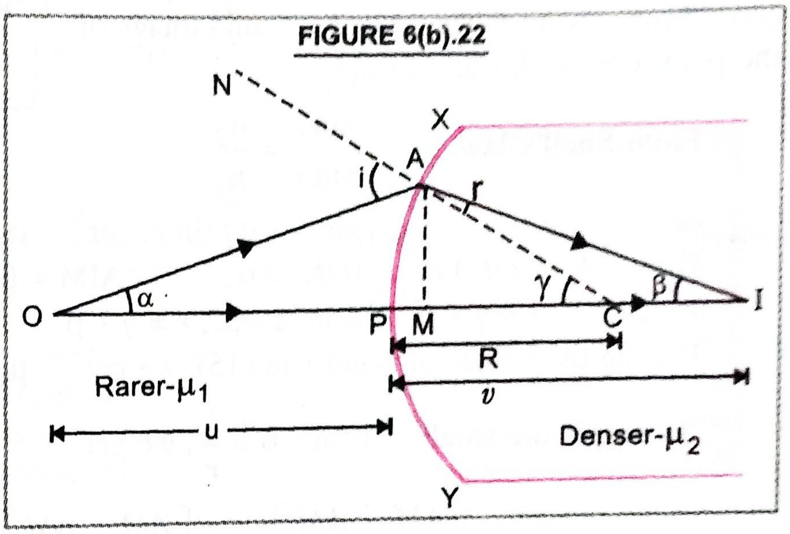


FIGURE 6(b).22

A ray of light starting from O and incident normally on the surface XY along OP passes straight. Another ray of light incident on XY along OA at $\angle i$ is refracted along AI at $\angle r$, bending towards the normal CAN. The two refracted rays actually meet at I, which is the real image of O.

From A, draw $AM \perp OI$.

Let $\angle AOM = \alpha$, $\angle AIM = \beta$

and $\angle ACM = \gamma$

As external angle of a triangle is equal to sum of internal opposite angles, therefore, in $\Delta\ IAC$

$$\gamma = r + \beta \quad \text{or} \quad \left.\begin{array}{l} r = \gamma - \beta \\ i = \alpha + \gamma \end{array}\right\} \qquad ...(10)$$

Similarly, in ΔOAC,

According to Snell's law, $$\frac{\mu_2}{\mu_1} = \frac{\sin i}{\sin r} \simeq \frac{i}{r} \quad (\because \text{ angles are small}) \qquad ...(11)$$

$$\therefore \quad \mu_1 i = \mu_2 r$$

Using (10), we get, $\mu_1 (\alpha + \gamma) = \mu_2 (\gamma - \beta)$

As angles α, β and γ are small, using $\theta = l/r$, we get

$$\therefore \quad \mu_1 \left(\frac{AM}{MO} + \frac{AM}{MC} \right) = \mu_2 \left(\frac{AM}{MC} - \frac{AM}{MI} \right) \qquad ...(12)$$

As aperture of the spherical surface is small, M is close to P. Therefore, $MO \approx PO$, $MI \approx PI$, $MC \approx PC$

From (12), $$\mu_1 \left(\frac{1}{PO} + \frac{1}{PC} \right) = \mu_2 \left(\frac{1}{PC} - \frac{1}{PI} \right) \quad \therefore \quad \frac{\mu_1}{PO} + \frac{\mu_2}{PI} = \frac{\mu_2 - \mu_1}{PC}$$

Using new cartesian sign conventions, we put

$$PO = -u,\ PI = +v,\ PC = R$$

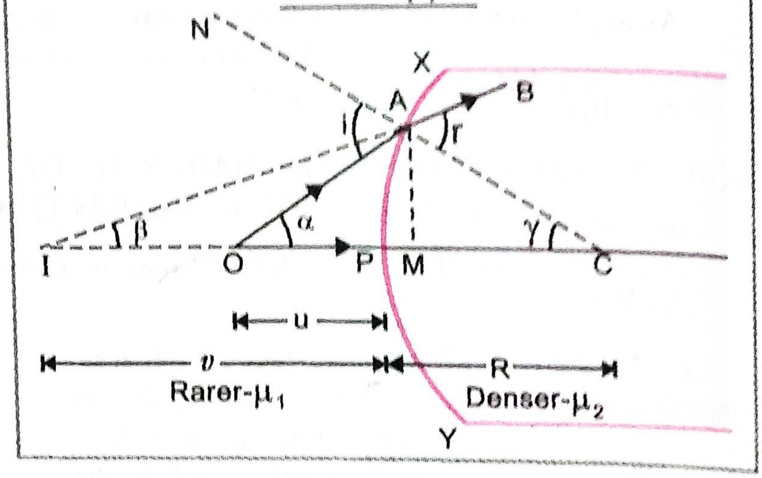
$$\boxed{\frac{\mu_1}{-u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}} \quad ...(13)$$

This is the relation governing refraction from rarer to denser medium at a convex spherical refracting surface.

(b) Virtual Image

The point object O in this case, lies on the principal axis *close* to the pole of the refracting surface. A ray of light OA starting from O suffers refraction at A, bends towards the normal CAN and is refracted along AB. Another ray of light OP falling normally on the refracting surface travels along PC undeviated. The refracted rays AB and PC do not meet actually at any point but appear to come from a point I. Thus I is the virtual image of the point object O, Fig. 6(b).23.

FIGURE 6(b).23



From Snell's law, $\frac{\sin i}{\sin r} = \frac{\mu_2}{\mu_1}$ ...(14)

or $\mu_1 \sin i = \mu_2 \sin r$ or $\mu_1 i = \mu_2 r$ ...(15) ($\because$ i and r are small)

Draw $AM \perp OI$. Let $\angle AOM = \alpha$, $\angle AIM = \beta$ and $\angle ACM = \gamma$

In $\Delta\ OAC$, $i = \gamma + \alpha$ and in $\Delta\ IAC$, $r = \gamma + \beta$

Putting the values of i and r in (15), we get $\mu_1 [\gamma + \alpha] = \mu_2 [\gamma + \beta]$

As angles are small, using $\theta = \frac{l}{r}$, we get,

or $\mu_1 \left[\frac{AM}{MC} + \frac{AM}{MO}\right] = \mu_2 \left[\frac{AM}{MC} + \frac{AM}{MI}\right]$ or $\mu_1 \left[\frac{1}{MC} + \frac{1}{MO}\right] = \mu_2 \left[\frac{1}{MC} + \frac{1}{MI}\right]$

As aperture is small, M is close to P

$\therefore$ $\mu_1 \left[\frac{1}{PC} + \frac{1}{PO}\right] = \mu_2 \left[\frac{1}{PC} + \frac{1}{PI}\right]$ or $\left[\frac{\mu_1}{PO} - \frac{\mu_2}{PI}\right] = \frac{\mu_2 - \mu_1}{PC}$

Using new cartesian sign conventions, $PO = -u$, $PI = -v$, $PC = +R$

$\frac{\mu_1}{-u} - \frac{\mu_2}{-v} = \frac{\mu_2 - \mu_1}{R}$ or $\boxed{-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$ ...(16)

Note that this relation is the same as equation (13), when image formed is real.

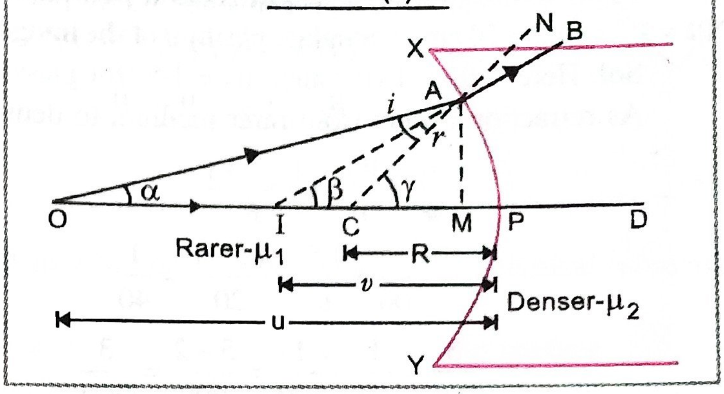
RETAIN IN MEMORY

Real images form on the side of a spherical refracting surface that is opposite the object ; and virtual images form on the same side as the object.

6(b).11. REFRACTION FROM RARER TO DENSER MEDIUM AT A CONCAVE SPHERICAL REFRACTING SURFACE

Let O be a point object lying on the principal axis of a spherical refracting surface XY which is concave towards the rarer medium. A ray of light OA starting from O suffers refraction at A, bends towards the normal CAN and is refracted along AB. Another ray of light OP, falling normally on the refracting surface suffers no deviation and passes along PD. The two refracted rays AB and PD do not meet at any point on the right side but appear to come from a point I on the left side. Thus I is virtual image of the point object O, Fig. 6(b).24.

FIGURE 6(b).24



In $\Delta\, OAC$, $i = \gamma - \alpha$ and in $\Delta\, IAC$, $r = \gamma - \beta$

Putting these values of i and r in the relation

$\mu_1 i = \mu_2 r$, we have $\mu_1(\gamma - \alpha) = \mu_2(\gamma - \beta)$

As angles are small, using $\theta = l/r$, we get

$$\therefore \quad \mu_1\left[\frac{AM}{MC} - \frac{AM}{MO}\right] = \mu_2\left[\frac{AM}{MC} - \frac{AM}{MI}\right]$$

$$\text{or} \quad \mu_1\left[\frac{1}{MC} - \frac{1}{MO}\right] = \mu_2\left[\frac{1}{MC} - \frac{1}{MI}\right]$$

As M is close to P because of small aperture

$$\therefore \quad \mu_1\left[\frac{1}{PC} - \frac{1}{PO}\right] = \mu_2\left[\frac{1}{PC} - \frac{1}{PI}\right] \quad \text{or} \quad \frac{\mu_2}{PI} - \frac{\mu_1}{PO} = \frac{\mu_2 - \mu_1}{PC}$$

Using new cartesian sign conventions,

$$PO = -u,\ PI = -v,\ PC = -R$$

$$\frac{\mu_2}{-v} - \frac{\mu_1}{-u} = \frac{\mu_2 - \mu_1}{-R} \quad \text{or} \quad \boxed{-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}}$$

which is the desired relation.

IMPORTANT NOTE

We have taken the object as a point object lying on the principal axis of the refracting surface. However, when the object has a finite size, we can calculate the size of the image from the linear magnification produced by the spherical refracting surface.

The course of rays is as shown in Fig. 6(b).25.

Compare equation (16), *i.e.*, $-\frac{\mu_1}{u} + \frac{\mu_2}{v} = \frac{\mu_2 - \mu_1}{R}$

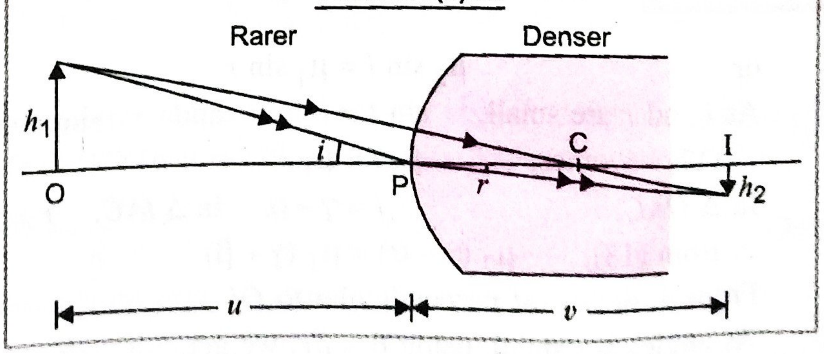
with the mirror equation

$$\frac{1}{u} + \frac{1}{v} = \frac{1}{f}$$

We find that u corresponds to $\left(\frac{-u}{\mu_1}\right)$

and v corresponds to $\frac{v}{\mu_2}$

FIGURE 6(b).25



$$\therefore \quad m = \frac{h_2}{h_1} = \frac{-v}{u} = -\frac{v/\mu_2}{-u/\mu_1}$$

$$\boxed{m = \frac{h_2}{h_1} = \frac{\mu_1 v}{\mu_2 u}}$$

This relation is valid for any single refracting surface, convex or concave.

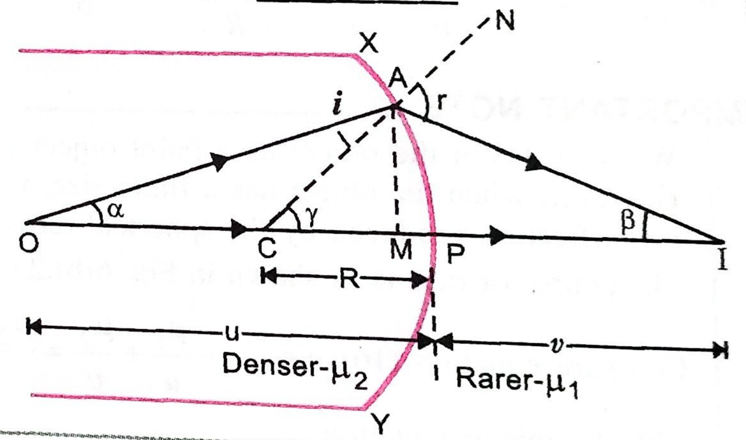
6(b).12. REFRACTION FROM DENSER TO RARER MEDIUM AT A SPHERICAL REFRACTING SURFACE

(a) At A convex spherical surface

Let P be the pole and C be the centre of curvature of a spherical refracting surface XY, Fig. 6(b).26. This surface is convex towards the rarer medium and separates a denser medium of refractive index μ_2 from a rarer medium of refractive index μ_1. Clearly, $\mu_2 > \mu_1$.

Let O be a point object lying on the principal axis of the spherical surface. A ray of light OA starting from O meets the refracting surface at A. On refraction, it bends away from the normal CAN and moves along AI. Another ray of light OP falling normally on the refracting surface goes undeviated along PI. The two refracted rays AI and PI actually meet at I, which is, therefore, the real image of the point object O. If i and r are the angles of incidence and refraction, then from Snell's law, we have

FIGURE 6(b).26



$$\frac{\mu_2}{\mu_1} = \frac{\sin r}{\sin i}$$

RETAIN IN MEMORY Note this step. Here, $\frac{\mu_2}{\mu_1} = \frac{\sin r}{\sin i}$ and not equal to $\frac{\sin i}{\sin r}$, because refraction occurs from denser to rarer medium.

or $$\mu_2 \sin i = \mu_1 \sin r \qquad ...(17)$$

As i and r are small, $\sin i \approx i$ and $\sin r \approx r$

$\therefore$ (17) becomes, $$\mu_2 i = \mu_1 r \qquad ...(18)$$

In $\Delta\ OAC$, $i = \gamma - \alpha$ In $\Delta\ IAC$, $r = \gamma + \beta$

$\therefore$ from (18), $\mu_2 (\gamma - \alpha) = \mu_1 (\gamma + \beta)$

From A, draw AM perpendicular to OI

As angles are small, using $\theta = l/r$, we get,

$$\therefore \quad \mu_2\left[\frac{AM}{MC} - \frac{AM}{MO}\right] = \mu_1\left[\frac{AM}{MC} + \frac{AM}{MI}\right] \quad \text{or} \quad \mu_2\left[\frac{1}{MC} - \frac{1}{MO}\right] = \mu_1\left[\frac{1}{MC} + \frac{1}{MI}\right]$$

Since aperture of the refracting surface is small, M is close to P.

$\therefore \quad MC \approx PC,\ MO \approx PO$ and $MI \approx PI \quad \therefore \quad \mu_2\left[\frac{1}{PC}-\frac{1}{PO}\right]=\mu_1\left[\frac{1}{PC}+\frac{1}{PI}\right]$

Applying new cartesian sign conventions, $PO = -u,\ PI = v,\ PC = -R$

$$\frac{\mu_2}{-u}+\frac{\mu_1}{v}=\frac{\mu_2-\mu_1}{-R}=\frac{\mu_1-\mu_2}{R} \qquad \text{...(19)}$$

This is the relation governing refraction from denser to rarer medium at a convex spherical refracting surface.

(b) Refraction from denser to rarer medium at a concave spherical refracting surface

Let O be a point object lying on the principal axis of a spherical refracting surface XY which is concave towards the rarer medium, Fig. 6(b).27. A ray of light OP falling normally on the surface goes undeviated along PC, whereas another ray OA, bends away from the normal and proceeds along AB after refraction. The refracted rays PC and AB do not actually meet at any point, but appear to come from a point I. Thus I is the virtual image of the point object O.

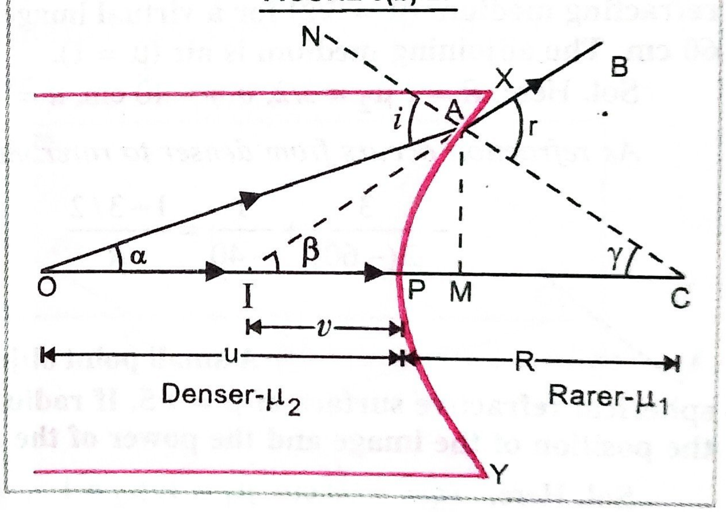
In $\Delta\ OAC$, $\quad i = \gamma + \alpha$

and in $\Delta\ IAC$, $\quad r = \gamma + \beta$

Putting these values of i and r in the relation $\mu_2 i = \mu_1 r$, we have $\mu_2(\gamma+\alpha)=\mu_1(\gamma+\beta)$

As angles are small, using $\theta = l/r$, we get

FIGURE 6(b).27



or $\mu_2\left[\frac{AM}{MC}+\frac{AM}{MO}\right]=\mu_1\left[\frac{AM}{MC}+\frac{AM}{MI}\right]$

or $\mu_2\left[\frac{1}{MC}+\frac{1}{MO}\right]=\mu_1\left[\frac{1}{MC}+\frac{1}{MI}\right]$

Since aperture is small, M is close to P

$\therefore \quad \mu_2\left[\frac{1}{PC}+\frac{1}{PO}\right]=\mu_1\left[\frac{1}{PC}+\frac{1}{PI}\right]$

or $\quad \frac{\mu_2}{PO}-\frac{\mu_1}{PI}=-\frac{\mu_2}{PC}+\frac{\mu_1}{PC}=\frac{-(\mu_2-\mu_1)}{PC}$

Using new cartesian sign conventions, so that $\quad PO = -u,\ PI = -v,\ PC = R$

$$\frac{\mu_2}{-u}-\frac{\mu_1}{-v}=\frac{-(\mu_2-\mu_1)}{R} \quad \text{or} \quad \frac{-\mu_2}{u}+\frac{\mu_1}{v}=\frac{\mu_1-\mu_2}{R} \qquad \text{...(20)}$$

which is the same as (19).